Intro to Rational Analysis and Probabilistic Models

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[based on slides by Sharon Goldwater & Frank Keller]

Mechanistic vs. Rational
- Mechanistic Modeling
- Rational Analysis

Example: Memory Retrieval
- Phenomena
- Rational Analysis
- Formalization
- Discussion

General discussion

Reading: Anderson, 1991; Chater and Oaksford, 1999.

Mechanistic Modeling

Traditional *mechanistic approach* to cognitive modeling (Chater and Oaksford, 1999):

- analyze cognitive phenomena (memory, reasoning, language) regarding their causal structure;
- stipulate architectures and algorithms;
- develop either symbolic or connectionist computational models;
- experimental and neuroscientific data provide constraints on these models.

Mechanistic Modeling

Problems with the mechanistic approach:

- cognitive systems are seen as an assortment of arbitrary mechanisms;
- they are subject to arbitrary constraints;
- the purpose or *goal structure* of the cognitive systems is left unexplained;
- the fact that cognitive systems are well adapted to the task they are solving and the environment they operate in is left unexplained.

Rational Analysis

Alternative: *Rational Analysis* approach to cognitive modeling:

- provide *purposive* explanations: analyze cognitive system as to its goal and function;
- specify the *task* a cognitive system solves and the nature of its *environment;* assume the system is optimally adapted to task and environment;
- derive an *optimal (rational) solution* to the task, subject to constraints (resource limitations);
- historically, this approach is related to probability theory; Bayesian mathematics often used to formulate models.

Rational Analysis and Probability Theory

Probability theory deals with making optimal decisions under uncertainty.

- making *predictions* about future events
- based on noisy input and incomplete information

Note similarities to machine learning! (Learning = generalization).

Rational Analysis

Methodology (Anderson, 1990, 1991):

1. **Goals:** specify precisely the goals of the cognitive system.
2. **Environment:** develop a formal model of the environment to which the systems is adapted.
3. **Computational Limitations:** make minimal assumptions about the computational limitations.
4. **Optimization:** derive the optimal behavior function, given (1)–(3).
5. **Data:** examine the empirical evidence to see whether the predictions of the behavior function are confirmed.
6. **Iteration:** repeat (1)–(5); iterative refinement.

Memory Retrieval

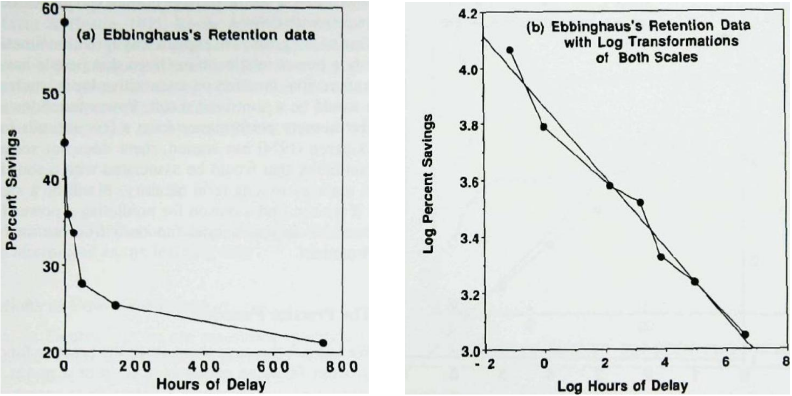
Why are some items retained and others not? Why decay over time?

- Traditional explanation: short term vs. long term store, memory is imperfect.
- Alternative explanation: keeping something in memory is costly and recent items are more likely to be needed again soon (ex: I'm likely to use *memory* and *retrieval* again soon).
- The memory system is optimally adapted to this decline in *need probability* over time.

Second explanation predicts that the shape of the forgetting function in memory should match the need probability in real situations.

Power Law of Forgetting

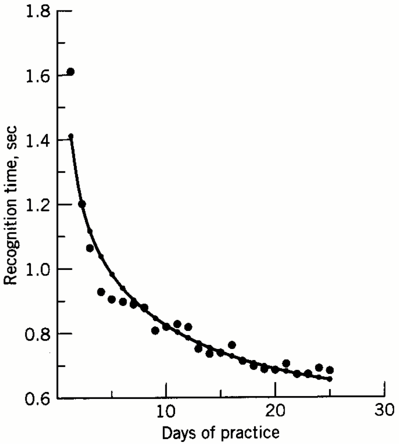
Proportion P of items retained at time t: $P = at^{-k}$, a *power-law* function. Appears linear when plotting $log(t)$ vs. $log(P)$.



Images: (Anderson and Schooler, 1991)

Power Law of Practice

Similar function holds between number of practice trials and reaction time (i.e., more frequent items recalled faster).



Rational Analysis of Memory Retrieval

1. **Goals:** efficient retrieval of items in memory; specifically: availability of an item should match the probability that it will be needed.
2. **Environment:** need-probability p for an item is determined by the environment; items with high p should be most available.
3. **Computational Limitations:** items are searched sequentially, with a fixed cost C with searching each item.
4. **Optimization:** stop retrieving items when $pG < C$, where G is the gain associated with retrieving an item; p depends on current context and item's history of use.

Rational Analysis of Memory Retrieval

5. **Data:** need to account for power law of forgetting, power law of practice.
6. **Iteration:** experiments that test the model:
    - investigate the role of context: recurrence of items in newspaper headlines;
    - manipulate need-probability experimentally; measure change in forgetting curves.

Formalization

Anderson, 1991 proposes that the need-probability p of an item A depends on its *history of use* H_A and the set of *contextual cues* Q that are present:

$$p = P(A|H_A, Q)$$

Assuming that the cues are independent of the history given A,

$$p \quad \propto \quad P(A|H_A)P(Q|A)$$

- $P(A|H_A)$: probability that A will be needed given its usage history;
- $P(Q|A)$: probability of observing the cues when A is needed (strength of association between A and Q).

History factor

Anderson (1991) assumes the same relationship between need-prob and history as found by Burrell (1980) when studying library borrowings. Specifically, $P(A|H_A)$

- decreases as a power function of time t since last use:

$$P(A|H_A) \propto t^{-k}$$

- increases as a power function of number of previous uses n.
- is maximized when t is equal to the interval between previous two uses.

All of these match subjects' memory behaviour, suggesting optimization for need-prob.

Context factor

Holding history constant, need-prob. is proportional to $P(Q|A)$.

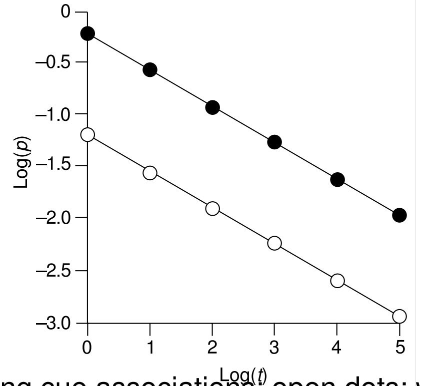
- $P(Q|A)$ is a product of separate cue strengths $P(q_i|A)$.
- Strength of cue i depends on direct association with A and association with items similar to A.

Model predicts various effects, including

- Memories are more accessible in the presence of related elements (priming).
- More subtle effects of prime frequency, number of related elements, etc.

Predictions

Relationship betw. need probability p and retention interval t:



Filled dots: strong cue associations; open dots: weak cue associations.

(Chater and Oaksford, 1999)

Discussion

- Controversy about power laws: can arise as an artifact of averaging over subjects.
  - But, evidence that power laws of forgetting and practice also hold for individual subjects.
- Experimental evidence for both context and history factors;
- Some effects (e.g. primacy) are not predicted by the model.
  - Need to take into account underlying mechanism (capacity of short-term memory).
  - Attempts to integrate cognitive architectures with rational explanations (ACT-*R*).

General discussion: Rational or irrational?

Many experiments conclude that people are ‘irrational’.

- **Decision-making**: subjects don’t follow rules of probability (*base rate neglect*).
- **Deductive reasoning**: subjects don’t follow rules of logic (*Wason selection task*).

But these experiments fail to account for people’s normal environment.

- Behavior comes much closer to normative rules when probabilities are *experienced* (vs. told) or rules are framed in real-world scenarios (vs. letters and numbers).
- Experiments often assume information is certain; real world is uncertain.

Adaptive rationality

Rational analysis assumes organisms are adapted to real world environments.

- Behavior is optimized over a range of situations, and given certain costs.
- Behavior may be non-optimal in specific situations (experiments).
- Example: Choice of local optimum over global optimum for reinforcement.

'Irrational' behavior may be the result of unnatural or unusual situations.

Summary

- Traditional modeling approaches treat the cognitive system as a collection of arbitrary mechanisms, with arbitrary performance limitations;
- they don't explain why these mechanisms cope with a complex and changing environment;
- rational analysis provides such explanations: analyze the task that a cognitive system solves, and its adaptation to the environment;
- optimal behavior functions explain why cognitive mechanisms are the way they are; provide constraints on possible theories and predict new data;
- successfully applied to memory, categorization, and other tasks.

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