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RANKING ARGUMENTS AND THE FUSIONAL REDUCTION ALGORITHM

1. THE PROBLEM AND THE PROPOSAL.

(1) Evidence for a particular constraint ranking (i.e. the 'primary' set of data):
  (i) the **violation profile** of a set of candidates (where a candidate is an I/O pair);
  (ii) the designation of a particular candidate as **the desired optimum**.

Tableau 1: the violation profile of **three** candidates (**a**, **b**, **c**) with respect to **three** constraints (**1**, **2**, **3**). The number in a cell: the number of 'offending' structures of a particular candidate with respect to a particular constraint.

Tableau 1	**1**	**2**	**3**
a: <I, O_a>	1	1	1
b: <I, O_b>	2	0	2
c: <I, O_c>	1	2	0

Desired optimum: **a: <I, O_a>**

A first formulation of the problem:

(2) what are the necessary and sufficient ranking conditions enforced by the 'primary' set of data, i.e. what are the constraint rankings that satisfy it?

Based on **Tableau 1**, we generate the comparative **Tableau 2** below, containing **two** candidate comparisons: **a** (optimum) ~ **b** (suboptimum); **a** (optimum) ~ **c** (suboptimum)

Tableau 2	**1**	**2**	**3**
r_1: a~b	W	L	W
r_2: a~c	e	W	L

- **W: the constraint prefers the desired optimum (the Winner)**
  |optimum violations|<|suboptimum violations|
- **L: the constraint prefers the suboptimum (the Loser)**
  |optimum violations|>|suboptimum violations|
- **e**: **the constraint does not distinguish between the compared candidates**
  |desired optimum violations|=|suboptimum violations|

A comparative tableau encodes the requirements that the 'primary' set of data enforces on the constraint ranking of the language under consideration:

*(3) A constraint ranking **satisfies** a comparative tableau **iff***
for each tableau row and constraint that assesses an L,
there is some constraint that: (a) assesses a W in that same row;
(b) dominates the L-assessing constraint.

e.g. **Tableau 2** is satisfied only by **1>>2>>3**:

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r_2 requires **2>>3**.
r_1 requires 1>>2 OR 3>>2.
But since 2>>3 holds (by r_2), 3>>2 is false; hence, we infer from r_1 and r_2 that **1>>2**.

Given that **a comparative tableau is a (partial) characterization of the constraint ranking of a particular language…**

THE PROBLEM:

(4) what are the necessary and sufficient ranking conditions enforced by a tableau, i.e. what are the constraint rankings that satisfy it?

(5) **Tableau 2** does not provide a perspicuous way to answer this question:
- the ranking of constraints 1 and 2 is not explicitly displayed;
- the ranking of constraints 1 and 3 is not explicitly displayed.

Tableau 2	**1**	**2**	**3**
r_1: a~b	W	L	W
r_2: a~c	e	W	L

(6) **Tableau 3** goes halfway towards this goal: 1>>2 is explicitly displayed (r_3).
Tableau 4 gives a complete answer: 1>>2 and 1>>3 are explicitly displayed (r_4).

Tableau 3	**1**	**2**	**3**
r_3: d~e	W	L	e
r_2: a~c	e	W	L

Tableau 4	**1**	**2**	**3**
r_4: f~g	W	L	L
r_2: a~c	e	W	L

(7) **Ranking conditions and comparative tableaux:**
  (a) **W**'s introduce **disjunctions**, e.g. r_1=WLW requires 1>>2 OR 3>>2
  (b) **L**'s introduce **conjunctions**, e.g. r_4=WLL requires 1>>2 AND 1>>3

(8) The problem becomes more pressing when the number of constraints and the number of comparisons under consideration is larger – see Chickasaw (pp. 13-14).

THE PROPOSED SOLUTION:

(9) For each comparative tableau, there is a unique canonical comparative tableau called **Maximally Informative Basis (MIB)** that:
  (a) is **equivalent** to it;
  (b) contains a **minimal number** of **independent** rows;
  (c) displays in a **perspicuous / maximally informative** way all the necessary and sufficient ranking conditions.

(10) We give **an algorithm** called **Fusional reduction (FRed)** that:
  (a) outputs the **MIB** for any input tableau;
  (b) can identify if the input tableau is contradictory and the minimal set of rows in the input tableau that yields a contradiction;
  (c) can perform Recursive Constraint Demotion (**RCD**).

Example: see Chickasaw (pp.13-14).

2. THE FUSIONAL REDUCTION ALGORITHM FOR OBTAINING THE MIB.

In this section, we will incrementally construct and motivate the steps of the algorithm.

Consider again **Tableaux 2**, **3** and **4** below.

Tableau 2	**1**	**2**	**3**
r_1: a~b	W	L	W
r_2: a~c	e	W	L

Tableau 3	**1**	**2**	**3**
r_3: d~e	W	L	e
r_2: a~c	e	W	L

Tableau 4	**1**	**2**	**3**
r_4: f~g	W	L	L
r_2: a~c	e	W	L

(11) **Question:** how do we arrive at **Tableau 4** starting from data that gives less informative tableaux like **Tableau 2** or **Tableau 3**?

(12) **Answer:** (a) ***FUSE*;**
(b) ***RETAIN ALL THE ROWS WHERE INFORMATION IS LOST IN THE FUSION***.

(13) **Fusion** (see Prince 2002a, b):

Fusion	**L**	**e**	**W**
L	L	L	L
e	L	e	W
W	L	W	W

a. **L** is dominant: for any X in {W, e, L}, f(L, X)=L;
b. **e** is identity: for any X in {W, e, L}, f(e, X)=X;
c. f(W, W)=W;
d. fusion of two rows r_1 and r_2 is obtained by **constraintwise** fusion and is abbreviated as: f(r_1, r_2).

Example: Row r_4 in **Tableau 4** is precisely the fusion of **Tableau 2** or **3**.

Tableau 2	**1**	**2**	**3**
r_1: a~b	W	L	W
r_2: a~c	e	W	L

	1	**2**	**3**
f(r_1, r_2)=r_4	**W**	**L**	**L**

Tableau 3	**1**	**2**	**3**
r_3: d~e	W	L	e
r_2: a~c	e	W	L

	1	**2**	**3**
f(r_3, r_2)=r_4	**W**	**L**	**L**

(14) The fusion f(r_1, r_2)=f(r_1, r_3) retains all the information in r_1 or r_3 and 'strengthens' it, i.e. it 'locally' **maximizes information** in rows r_1 and r_3 based on the rest of the tableau.

(15) **Q:** What exactly is **'strengthening' / 'maximizing' information** in a row?
(16) **A:** Obtaining another row that **entails** it.

(17) **<u>Entailment:</u>** A tableau T_1 entails a tableau T_2 iff

{constraint rankings satisfying T_1}$\subseteq${constraint rankings satisfying T_2}

e.g.

Tableau 4	**1**	**2**	**3**
r_4: f~g	W	L	L
r_2: a~c	e	W	L

T_2	**1**	**2**	**3**
	W	L	e
	W	e	L

Tableau 4 entails **T_2** because:
{1>>2>>3} satisfies **Tableau 4**;
{1>>2>>3, 1>>3>>2} satisfies **T_2**.

(18) **However: fusion is not enough!**
The ranking information provided by r_2=eWL, i.e. 2>>3, is lost in the fusion f(r_1, r_2)=WLL, which requires 1>>2 AND 1>>3.

Tableau 2	**1**	**2**	**3**
r_1: a~b	W	L	W
r_2: a~c	e	W	L

	1	**2**	**3**
f(r_1, r_2)=r_4	**W**	**L**	**L**

(19) What we want is to obtain an **equivalent** tableau. **Hence:** we **keep row r_2 together with the fusion f(r_1, r_2)** – which yields precisely **Tableau 4**!

Tableau 2	**1**	**2**	**3**
r_1: a~b	W	L	W
r_2: a~c	e	W	L

	1	**2**	**3**
f(r_1, r_2)	**W**	**L**	**L**

	1	**2**	**3**
r_2	**e**	**W**	**L**

(20) **<u>Equivalence:</u>** A tableau T_1 is equivalent with a tableau T_2 iff
{constraint rankings satisfying T_1}={constraint rankings satisfying T_2}

e.g.

Tableau 2	**1**	**2**	**3**
r_1: a~b	W	L	W
r_2: a~c	e	W	L

Tableau 3	**1**	**2**	**3**
r_3: d~e	W	L	e
r_2: a~c	e	W	L

Tableau 4	**1**	**2**	**3**
r_4: f~g	W	L	L
r_2: a~c	e	W	L

<u>The general problem is:</u>

(21) **Q:** How do we identify the rows that lose information in the fusion?
(22) **A:** *By identifying* ***info loss configurations***.

(23) **Only one case in which no information is lost in the fusion:** when all the W's in the fusion come only from the fusion of W's (i.e. no e's).

Example: Consider **Tableau 11** below. The W in the fusion comes only from the fusion of W's and the fusion **f(r_3, r_6)** is indeed equivalent with the two fused rows.

Tableau 11	**1**	**2**	**3**
r_3	W	L	e
r_6	W	e	L
f(r_3, r_6)	**W**	**L**	**L**

r_3: 1>>2; **r_6:** 1>>3 **f(r_3, r_6):** 1>>2 AND 1>>3

(24) In **Tableau 2**, the fusion lost information because the W in the fusion did not come only from the fusion of W's.

Tableau 2	**1**	**2**	**3**
r_1: a~b	W	L	W
r_2: a~c	e	W	L
f(r_1, r_2)	**W**	**L**	**L**

(25) ***Info loss configuration*** **:=** a constraint **i** in a tableau that contains only **e**'s and at least one **W**.

Example: constraint **1** in **Tableau 2** is the only **info loss configuration** in that tableau.

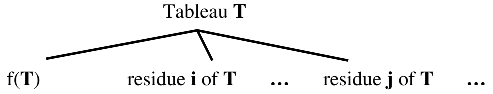
(26) ***Info residue*** **:=** for every info loss configuration **i**, the info residue is the set of rows that have an **e** in that info loss configuration.

Example: the **info residue** corresponding to info loss configuration **1** in **Tableau 2** is r_2=eWL.

(27) Intermediate summary 1 – to obtain the MIB do the following:

- **A. Whole fusion:**
  fuse all tableau rows and construct a branch for the fusion.
- **B. Info Loss:**
  identify all the info loss configurations; for each info loss configuration, construct a branch with the info residue.

(28) The general schema is given in the tree below:



Example:

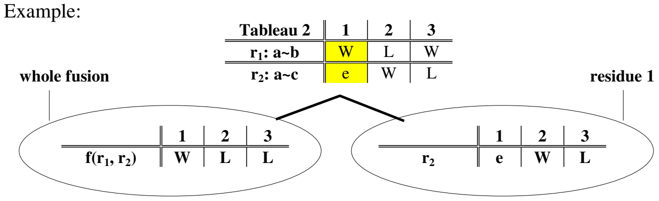


Tableau 2	**1**	**2**	**3**
r_1: a~b	W	L	W
r_2: a~c	e	W	L

	1	**2**	**3**
f(r_1, r_2)	**W**	**L**	**L**

	1	**2**	**3**
r_2	**e**	**W**	**L**

(29) (a) the root of the tree, i.e. **Tableau 2**, is equivalent to its daughters.
(b) the two daughters put together form **Tableau 4**.

(30) A more complex example is provided by **Tableau 12** below: we **recursively** apply the general schema in (28) to all the daughters, until there are no more info loss configurations.

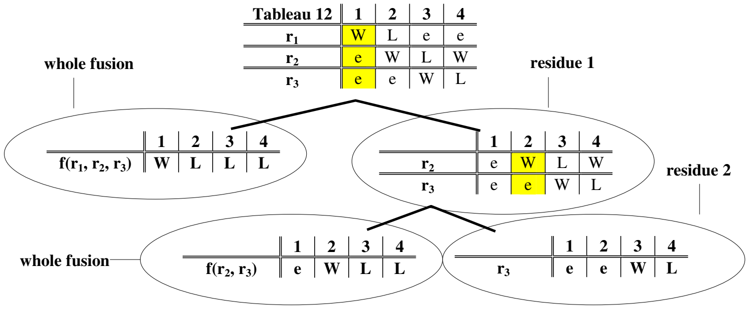


Tableau 12	**1**	**2**	**3**	**4**
r_1	W	L	e	e
r_2	e	W	L	W
r_3	e	e	W	L

	1	**2**	**3**	**4**
f(r_1, r_2, r_3)	**W**	**L**	**L**	**L**

	1	**2**	**3**	**4**
r_2	e	W	L	W
r_3	e	e	W	L

	1	**2**	**3**	**4**
f(r_2, r_3)	**e**	**W**	**L**	**L**

	1	**2**	**3**	**4**
r_3	**e**	**e**	**W**	**L**

(31) Each node of the tree is equivalent to its daughters.

(32) New info loss configurations can appear at the intermediate levels of the tree: constraint **2** is **not** an info loss configuration **at the root level**, but it **is** an info loss configuration at the intermediate level (**residue 1**).

(33) The dynamics of 'disclosing' new info loss configurations is common to **FRed** and **RCD**.

(34) **To obtain the MIB, we just have to collect all the terminal nodes of the tree** – as in **Tableau 13** below.

Tableau 13	**1**	**2**	**3**	**4**
f(r_1, r_2, r_3)	W	L	L	L
f(r_2, r_3)	e	W	L	L
r_3	e	e	W	L

(35) Intermediate summary 2 – to obtain the MIB do the following:

CORE:

CoreA. Whole fusion:
fuse all tableau rows and construct a branch for the fusion.

CoreB. Info Loss:
identify all the info loss configurations; for each info loss configuration, construct a branch with the info residue.

RECURSION:

RecurseA. Recursion:
for every info residue, apply steps (**CoreA-C**).

RecurseB. Terminate:
continue until there are no info residues.

RecurseC. Collect:
collect all the terminal nodes: this is **MIB**(input tableau).

(36) Applying this procedure:
- (a) we obtain a final tableau that is **equivalent** to the initial tableau;
- (b) we **maximize informativeness** in each row of the output tableau;
- (c) we make the resulting tableau **more concise**.

(37) To see that the resulting tableau is as concise as possible, consider again **Tableau 11**. There is **no info loss configuration**, hence we construct only the fusion branch and the resulting tableau contains only one row (as opposed to two).

Tableau 11	**1**	**2**	**3**
r_3	W	L	e
r_6	W	e	L

	1	**2**	**3**
f(r_3, r_6)	**W**	**L**	**L**

(38) **However: fusion is not always useful!** Consider **Tableau 14** below: we construct a branch for each info loss configuration and store the corresponding info residue.

(39) **Fusion is futile in this case: f(r_6, r_2)** is entailed by each of the fused rows **r_6** and **r_2**. Hence: **we should discard the fusion when we collect the terminal nodes**.

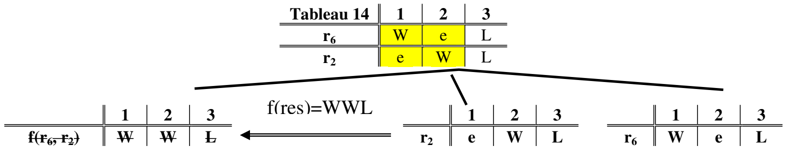


Tableau 14	**1**	**2**	**3**
r_6	W	e	L
r_2	e	W	L

	1	**2**	**3**
~~f(r_6, r_2)~~	~~W~~	~~W~~	~~L~~

f(res)=WWL

	1	**2**	**3**
r_2	**e**	**W**	**L**

	1	**2**	**3**
r_6	**W**	**e**	**L**

(40) The **MIB** of **Tableau 14** is **Tableau 14** itself!

(41) **Hence:** we should add an **Entailment Check** step in the procedure given in (35), so that we eliminate futile fusions, i.e. **we should check whether the residues entail the fusion of the whole**.

(42) **<u>Entailment check:</u>** it (can be proved that it) is sufficient to check whether the fusion of the all the residues entails the fusion of the whole mother node, i.e.

Check: f(All Residue Daughters) entails f(Whole mother)?
If yes, discard f(Whole mother); else, keep it.

(43) THE FUSIONAL REDUCTION (FRED) ALGORITHM:

CORE:

CoreA. Whole fusion:
fuse all tableau rows and construct a branch for the fusion.

CoreB. Info Loss:
identify all the info loss configurations; for each info loss configuration, construct a branch with the info residue.

CoreC. Entailment check:
fuse of all the rows in the info residues constructed in (**CoreB**); if the fusion of residues entails the whole fusion in (**CoreA**), discard the whole fusion in (**CoreA**), else keep it.

RECURSION:

RecurseA. Recursion:
for every info residue, apply steps (**CoreA-C**).

RecurseB. Terminate:
continue until there are no info residues.

RecurseC. Collect:
collect all the terminal nodes: this is **MIB**(input tableau).

2.1. APPLYING FRED: A MORE COMPLEX EXAMPLE.

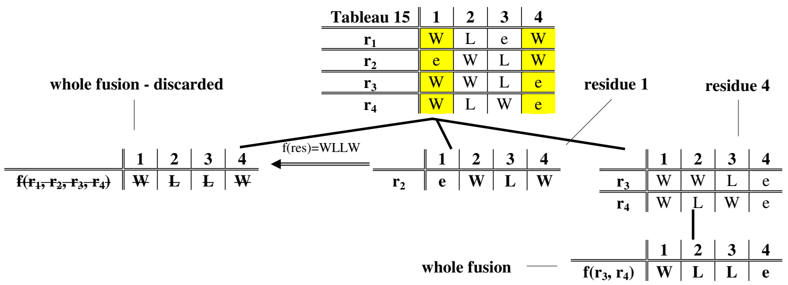


Tableau 15	1	2	3	4
r_1	W	L	e	W
r_2	e	W	L	W
r_3	W	W	L	e
r_4	W	L	W	e

	1	2	3	4
~~$f(r_1, r_2, r_3, r_4)$~~	~~W~~	~~L~~	~~L~~	~~W~~

	1	2	3	4
r_2	e	W	L	W

	1	2	3	4
r_3	W	W	L	e
r_4	W	L	W	e

	1	2	3	4
$f(r_3, r_4)$	W	L	L	e

(44) Applying **FRed** to **Tableau 15**:

CoreA. Whole fusion: we obtain $f(r_1, r_2, r_3, r_4)$=WLLW.

CoreB. Info Loss: there are two info loss configurations: constraint **1** and **4**; for each of them, we construct a branch with the info residue, i.e. residue**1={r_2}** and residue**4={r_3, r_4}**.

CoreC. Entailment check: the fusion of all the info residues is f(res)=WLLW, which is identical to $f(r_1, r_2, r_3, r_4)$ – hence, we discard $f(r_1, r_2, r_3, r_4)$.

RecurseA. Recursion: for every info residue, we apply step (**a**):
residue**1** – $f(r_2)=r_2$ (trivially)
residue**4 –** $f(r_3, r_4)$=WLLe;
there are no more info loss configurations, hence steps (**CoreB**, **CoreC**) do not apply.
RecurseB. Terminate: no more info residues.
RecurseC. Collect: all the terminal nodes are collected in **Tableau 16** below;
this is the **MIB**.

Tableau 16	1	2	3	4
r_2	e	W	L	W
$f(r_3, r_4)$	W	L	L	e

(45) The above example is interesting because:

- we have another example of **reduction / conciseness:** we eliminate the redundant / entailed row r_1 and we collapse rows r_3 and r_4 (that do not create any info loss configuration) into one row that entails both of them.
- we see how a fusion is discarded due to **Entailment Check** (step **CoreC**);
- we see a tree with more than two levels;
- we see an instance of ternary branching, because there are **two** info loss configurations and **two** corresponding residues.

2.2. PROPERTIES OF THE FRED.

- **FRed eliminates entailed rows.**
- **FRed collapses all the rows that do not create any info loss configuration (i.e. W-compliant rows – see Prince 2002a)**
- **FRed incorporates maximal information into each derived row.**
- **FRed and Recusive constraint demotion (RCD):** the algorithm can perform RCD – instead of constructing a branch for each info residue (**CoreB**), intersect all the info residues and construct **only one branch for this intersection**; the fusions that are collected at the end are the fusions based on which RCD extracts the constraint ranking.
- **FRed detects inconsistency:** if it is applied to an inconsistent (contradictory) tableau (see (46) below), then the output of the algorithm is going to contain a row with at least one L and no W's (hence, an unsatisfiable row).
- **FRed detects the maximal set of inconsistent rows:** for an inconsistent tableau, the output of FRed contains a row with at least one L and no W's; this row appears at some node in the tree; the mother of that node is the corresponding maximal set of inconsistent rows.

(46) **A contradictory / inconsistent tableau** is not satisfied by any constraint ranking.

Contradictory	1	2	3
r_1	L	W	e
r_2	W	L	e
$f(r_1, r_2)$	L	L	e

3. CONCLUSION.

The present paper posed **the following question**: given that a comparative tableau is a (partial) characterization of the constraint ranking of a particular language, how can we know:

(4) what are the necessary and sufficient ranking conditions enforced by a tableau, i.e. what are the constraint rankings that satisfy it?

The proposed solution: for each (non-contradictory) comparative tableau T, there is a unique canonical tableau called **Maximally Informative Basis (MIB)** such that:

(a) MIB(T) is **equivalent** to T;
(b) MIB(T) contains a **minimal number** of **independent** rows;
(c) each row in **MIB**(T) is **maximally informative**.

The **MIB** answers question **(4)**: it displays in a **perspicuous / maximally informative** way all the necessary and sufficient ranking conditions.

Finally, **the Fusional Reduction (FRed) algorithm** outputs the **MIB** for any (non-contradictory and non-trivial) input tableau. In addition, **FRed**:

(a) can identify if the input tableau is contradictory and the minimal set of rows in the input tableau that yields a contradiction;
(b) can perform Recursive Constraint Demotion (**RCD**).

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CHICKASAW ERC SET (ADRIAN BRASOVEANU, 2002) – 14 CONSTRAINTS, 41 ROWS:

X	**C1**	**C2**	**C3**	**C4**	**C5**	**C6**	**C7**	**C8**	**C9**	**C10**	**C11**	**C12**	**C13**	**C14**
1	W	W	e	L	W	W	L	e	e	W	W	e	e	e
2	e	e	e	L	W	e	L	W	W	W	W	e	e	e
3	e	W	e	e	e	W	e	L	e	e	e	e	e	e
4	e	W	e	e	W	e	L	L	e	W	W	e	e	e
5	e	e	e	e	W	e	L	e	W	W	W	e	e	e
6	e	e	e	L	W	e	L	W	W	W	W	W	e	e
7	e	L	e	e	e	L	e	W	W	e	e	e	e	e
8	e	e	W	e	e	e	W	e	e	L	L	e	e	e
9	e	L	e	W	e	L	W	W	e	L	L	W	e	e
10	e	L	e	e	W	L	e	W	W	e	e	W	e	e
11	e	L	e	e	W	L	W	W	W	L	L	W	e	e
12	e	e	e	e	e	W	L	e	e	W	e	e	e	e
13	e	L	e	e	e	e	e	e	W	e	e	e	e	e
14	e	L	e	e	e	e	L	e	W	W	e	e	e	e
15	e	L	e	W	L	e	e	e	e	e	L	e	e	e
16	e	L	e	W	L	e	L	e	e	e	L	e	e	e
17	e	L	e	W	L	e	L	e	W	W	e	e	e	e
18	e	e	W	e	e	e	e	e	e	e	L	e	e	e
19	W	W	e	L	W	e	e	e	e	e	W	e	e	e
20	e	e	e	W	L	e	W	L	e	e	e	e	e	e
21	e	e	e	W	e	e	e	L	e	e	e	e	e	e
22	e	e	e	W	W	e	L	L	W	W	W	e	e	e
23	e	e	e	e	W	e	L	e	W	W	W	e	e	e
24	e	e	e	L	W	e	L	W	W	W	W	e	e	e
25	e	e	e	L	W	e	e	W	W	e	e	e	e	e
26	W	e	e	e	e	W	e	L	L	e	e	L	e	e
27	e	e	e	e	e	e	W	e	e	L	e	e	e	e
28	e	e	e	e	e	e	W	e	e	L	e	e	e	e
29	e	e	e	e	e	L	W	W	e	L	e	W	e	e
30	e	e	e	e	e	L	W	W	e	e	e	e	e	e
31	e	e	e	e	e	L	e	W	e	e	e	W	e	L
32	e	e	e	e	e	W	L	e	e	W	e	e	e	e
33	e	e	e	e	e	W	L	e	e	e	e	e	e	e
34	e	e	e	e	e	e	W	e	e	L	e	e	e	e
35	e	e	W	L	e	e	e	e	e	e	e	e	e	e
36	e	e	e	L	W	e	L	W	W	W	W	W	e	e
37	e	e	e	L	W	e	e	W	W	e	W	e	e	e
38	e	e	e	e	e	L	e	W	e	e	e	e	e	L
39	e	W	e	L	e	e	e	e	e	e	e	e	W	L
40	e	e	e	e	e	e	W	e	e	L	L	e	e	L
41	e	e	W	L	e	e	W	e	e	L	L	e	e	L

REDUCED TO MIB VIA THE IMPLEMENTATION OF FRED IN MERCHANT 2004 – 9 ROWS:

X	C1	C13	C3	C9	C4	C2	C8	C6	C7	C12	C5	C14	C10	C11
f1	**W**	*e*	*e*	L	L	L	L	L	L	L	L	L	L	L
f2	*e*	**W**	*e*	*e*	L	L	L	L	L	*e*	L	L	L	L
f3	*e*	*e*	**W**	*e*	L	L	L	L	L	*e*	L	L	L	L
f4	*e*	*e*	*e*	**W**	L	L	L	L	L	*e*	L	L	L	L
f5	*e*	*e*	*e*	*e*	**W**	L	L	L	L	*e*	L	L	L	L
f6	*e*	*e*	*e*	*e*	*e*	**W**	L	L	L	*e*	*e*	L	L	L
f7	*e*	*e*	*e*	*e*	*e*	*e*	**W**	L	L	*e*	*e*	L	L	L
f8	*e*	*e*	*e*	*e*	*e*	*e*	*e*	**W**	L	*e*	*e*	L	L	L
f9	*e*	*e*	*e*	*e*	*e*	*e*	*e*	*e*	**W**	*e*	*e*	L	L	L

Input data:	41 rows
MIB:	9 rows

Avg. disjunctivity of data:	2.7 W's/row	Max disjunctivity of data:	6 W's in one row
Avg. disjunctivity **MIB**:	1 W/row	Max disjunctivity **MIB**:	1 W/row
Avg. conjunctivity of data:	1.8 L/row	Max conjunctivity of data:	4 L's in one row
Avg. conjunctivity **MIB**:	7.1 L/row	Max conjunctivity **MIB**:	11 L's in one tow

Summary:

- nearly **5:1** tableau shrinkage;
- nearly **3:1** reduction in avg. disjunctivity;
- nearly **1:4** increase in avg. conjunctivity.