The Maximally Informative Basis and The Fusional Reduction Algorithm

The problem. In Optimality Theory, the language specific part of grammatical knowledge comprises a particular constraint ranking, i.e. a strict total ordering of the universal set of constraints. The evidence for such a constraint ranking, henceforth called 'the primary set of data', is provided by a tableau like (1) below, which consists of: **(a)** the **violation profile** of a set of candidates (where a candidate is an Input/Output pair); **(b)** the designation of a particular candidate as **the desired optimum**. Tableau 1 encodes the violation profile of **three** candidates (**a**, **b**, **c**) with respect to **three** constraints ($\mathbf{C_1}$, $\mathbf{C_2}$, $\mathbf{C_3}$). The number in a cell is the number of 'offending' structures of a particular candidate with respect to a particular constraint. To extract (partial) knowledge about the particular constraint ranking, the learner/linguist has to answer the question in (1) below:

(1) what are the necessary and sufficient ranking conditions enforced by the primary set of data, i.e. what are the constraint rankings that satisfy it?

The present paper: **(a)** shows that it is possible to answer question (1) for any given primary set of data (tableau) by providing its Maximally Informative Basis (MIB), which perspicuously displays the necessary and sufficient ranking conditions; and **(b)** gives an algorithm called Fusional Reduction (FRed) to obtain the MIB of an arbitrary initial tableau. The usefulness of the MIB and FRed becomes obvious as soon as one considers real-life analyses, which involve more than a couple of constraints and candidates.

The proposal. Previous research provided partial answers to question (1): the Recursive Constraint Demotion (RCD) algorithm in [5], [6] and [7] provides a sufficient, but generally not necessary, set of ranking conditions for an arbitrary initial tableau; the comparative tableau format and the operation of fusion introduced in [3] and [4] provide a procedure for extracting non-obvious necessary, but generally not sufficient, ranking conditions for arbitrary input tableaux. The present proposal builds on [3] and [4].

First, the comparative tableau format is used, so Tableau 1 is converted into the comparative Tableau 2 below (**W**: the constraint prefers the desired optimum; **L**: the constraint prefers the suboptimum; **e**: the constraint does not distinguish between the compared candidates). A constraint ranking is said to **satisfy a comparative tableau** iff for each tableau row and constraint that assesses an L, there is some constraint that: **(a)** assesses a W in that same row; and **(b)** dominates the L-assessing constraint. For example, Tableau 2 is satisfied only by $\mathbf{C_1{>>}C_2{>>}C_3}$. However, Tableau 2 does not provide a perspicuous way to answer question (1): it explicitly displays the ranking of constraints $\mathbf{C_2}$ and $\mathbf{C_3}$ (row $\mathbf{r_2}$ enforces $\mathbf{C_2{>>}C_3}$), but it does not explicitly display the ranking of constraints $\mathbf{C_1}$ and $\mathbf{C_2}$ (i.e. $\mathbf{C_1{>>}C_2}$) or $\mathbf{C_1}$ and $\mathbf{C_3}$ (i.e. $\mathbf{C_1{>>}C_3}$).

Second, we observe that Tableau 2 is equivalent to Tableau 3 and Tableau 4 below, since they are satisfied by the same constraint ranking. However, Tableau 3 goes halfway towards answering question (1), since row $\mathbf{r_3}$ explicitly displays the ranking of $\mathbf{C_1}$ and $\mathbf{C_2}$ (i.e. $\mathbf{C_1{>>}C_2}$); and Tableau 4 gives a complete answer to question (1), as row $\mathbf{r_4}$ explicitly shows the ranking of $\mathbf{C_1}$ and $\mathbf{C_3}$ (i.e. $\mathbf{C_1{>>}C_3}$) in addition to displaying $\mathbf{C_1{>>}C_2}$.

Tableau 4 is called the MIB of Tableau 2 (or, equivalently, of Tableau 1); the MIB answers question (1) because: **(a)** it is **equivalent** to the initial tableau; **(b)** it contains a **minimal number** of **independent** rows; and **(c)** it displays in a **perspicuous / maximally informative** way all the necessary and sufficient ranking conditions. The paper shows that the MIB exists and is unique for an arbitrary (consistent) initial tableau and that there is an algorithm based on the operation of fusion (i.e. FRed) which can compute the MIB of any tableau.

TABLEAUX:

Tableau 1	**C_1**	**C_2**	**C_3**
a: <I, O_a>	1	1	1
b: <I, O_b>	2	0	2
c: <I, O_c>	1	2	0

Desired optimum: **a: <I, O_a>**

Tableau 2	**C_1**	**C_2**	**C_3**
r_1: a~b	W	L	W
r_2: a~c	e	W	L

Tableau 3	**C_1**	**C_2**	**C_3**
r_3	W	L	e
r_2: a~c	e	W	L

Tableau 4	**C_1**	**C_2**	**C_3**
r_4	W	L	L
r_2: a~c	e	W	L

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