Monetary Theory and Policy, 2nd ed. MIT Press, 2003
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Chapter 6: Money and the Open Economy
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These notes provide a more detailed (and corrected) derivation of the model of section 6.5. In the text, equation (6.82) is incorrect (see 18 below) and this affects some of the subsequent expressions.

Section 6.5.1: A Basic Open-Economy Model

The basic model of section 6.5 results in the following equilibrium relationships:

Consumer price index:

$$p_t^c = (1-\gamma)p_t^h + \gamma p_t^f = p_t^h + \gamma\delta_t, \tag{1}$$

Aggregate consumption index:

$$c_t = (1-\gamma)c_t^h + \gamma c_t^f \tag{2}$$

Relative demand for foreign goods:

$$c_t^f = -a\delta_t + c_t^h \tag{3}$$

Euler condition:

$$c_t = \mathrm{E}_t c_{t+1} - \left(\frac{1}{\sigma}\right)\left(i_t - \mathrm{E}_t \pi_{t+1}^c\right) \tag{4}$$

Labor supply condition:

$$\eta n_t + \sigma c_t + \mu_t^w = w_t - p_t^c = w_t - p_t^h - \gamma\delta_t, \tag{5}$$

where $\pi_t^c = p_t^c - p_{t-1}^c$ and $\delta_t = \gamma(p_t^f - p_t^h)$ and μ_t^w is the Clarida, Galí, and Gertler (2001) stochastic *wage markup.*

Combining (2) and (3), $c_t = c_t^h - \gamma a\delta_t$. Defining $\pi_t^h = p_t^h - p_{t-1}^h$,

$$\pi_t^c = p_t^c - p_{t-1}^c = \pi_t^h + \gamma(\delta_t - \delta_{t-1}). \tag{6}$$

Production:

$$y_t^h = n_t + \varepsilon_t. \tag{7}$$

Marginal cost:

$$mc_t = w_t - p_t^h - \varepsilon_t. \tag{8}$$

The inflation rate for the price index of domestically produced goods is

$$\pi_t^h = \beta \mathrm{E}_t \pi_{t+1}^h + \kappa mc_t, \tag{9}$$

where $\kappa = (1-\omega)(1-\beta\omega)/\omega$.

The Foreign Country

Assume foreign households have the same preferences as those of the home country (so the demand elasticity is the same). Then,

$$c^{h_t^*} = a\delta_t + y_t^f, \tag{10}$$

where y_t^* is foreign income.

The Euler condition for foreign country households implies

$$y_t^f = \mathrm{E}_t y_{t+1}^f - \left(\frac{1}{\sigma}\right)\left(i_t^f - \mathrm{E}_t \pi_{t+1}^f\right),$$

or

$$\rho_t^f \equiv i_t^f - E_t \pi_{t+1}^f = \sigma\left(\mathrm{E}_t y_{t+1}^f - y_t^f\right). \tag{11}$$

Equilibrium Conditions

Home production equal the consumption of the domestically produced good:

$$y_t = (1-\gamma)c_t^h + \gamma c_t^{h^*}. \tag{12}$$

Using (2) and (3),

$$c_t = (1-\gamma)c_t^h + \gamma c_t^f = c_t^h - \gamma a\delta_t.$$

Now combining this with (10), (12) can be written as

$$\begin{aligned} y_t &= (1-\gamma)c_t^h + \gamma c_t^{h^*} = (1-\gamma)\left[c_t + \gamma a\delta_t\right] + \gamma a\delta_t + \gamma y_t^f \\ &= (1-\gamma)c_t + (2-\gamma)\gamma a\delta_t + \gamma y_t^f. \end{aligned} \tag{13}$$

Uncovered interest parity implies

$$i_t = i_t^f + \mathrm{E}_t s_{t+1} - s_t,$$

or in real terms,

$$\rho_t^h = i_t - \mathrm{E}_t\pi_{t+1}^h = \rho_t^f + \left(\mathrm{E}_t\delta_{t+1} - \delta_t\right) \Rightarrow \rho_t^h = \rho_t^f + \left(\mathrm{E}_t\delta_{t+1} - \delta_t\right). \tag{14}$$

But

$$i_t - \mathrm{E}_t\pi_{t+1}^c = \rho_t^h + \left(\mathrm{E}_t\pi_{t+1}^h - \mathrm{E}_t\pi_{t+1}^c\right) = \rho_t^h - \gamma\left(\mathrm{E}_t\delta_{t+1} - \delta_t\right),$$

so

$$i_t - \mathrm{E}_t\pi_{t+1}^c = \rho_t^h - \gamma\left(\mathrm{E}_t\delta_{t+1} - \delta_t\right) = \rho_t^f + (1-\gamma)\left(\mathrm{E}_t\delta_{t+1} - \delta_t\right).$$

Thus, the Euler condition (4) becomes

$$\begin{aligned} c_t &= \mathrm{E}_t c_{t+1} - \left(\frac{1}{\sigma}\right)\left[\rho_t^f + (1-\gamma)\left(\mathrm{E}_t\delta_{t+1} - \delta_t\right)\right] \\ &= \mathrm{E}_t c_{t+1} - \left(\frac{1}{\sigma}\right)\left[\rho_t^f + (1-\gamma)\left(\rho_t^h - \rho_t^f\right)\right] \\ &= \mathrm{E}_t c_{t+1} - \left(\frac{1}{\sigma}\right)\left[(1-\gamma)\rho_t^h + \gamma\rho_t^f\right]. \end{aligned}$$

This relationship between domestic output and consumption (13) can be used to eliminate c_t from the Euler condition (4), yielding

$$\begin{aligned} y_t^h &= \mathrm{E}_t y_{t+1}^h + (1-\gamma)(c_t - \mathrm{E}_t c_{t+1}) - (2-\gamma)\gamma a\left(\mathrm{E}_t\delta_{t+1} - \delta_t\right) - \gamma \mathrm{E}_t\left(y_{t+1}^f - y_t^f\right) \\ &= \mathrm{E}_t y_{t+1}^h - (1-\gamma)\left(\frac{1}{\sigma}\right)\left[(1-\gamma)\rho_t^h + \gamma\rho_t^f\right] - (2-\gamma)\gamma a\left(\rho_t^h - \rho_t^f\right) - \gamma \mathrm{E}_t\left(y_{t+1}^f - y_t^f\right) \\ &= \mathrm{E}_t y_{t+1}^h - (1-\gamma)\left(\frac{1}{\sigma}\right)\left[(1-\gamma)\rho_t^h + \gamma\rho_t^f\right] - (2-\gamma)\gamma a\left(\rho_t^h - \rho_t^f\right) - \frac{\gamma}{\sigma}\rho_t^f \end{aligned}$$

or

$$\begin{aligned} y_t^h &= \mathrm{E}_t y_{t+1}^h - \left(\frac{1}{\sigma}\right)\left[(1-\gamma)^2 + (2-\gamma)\gamma\sigma a\right]\rho_t^h - \left(\frac{1}{\sigma}\right)\left[(1-\gamma)\gamma - (2-\gamma)\gamma\sigma a + \gamma\right]\rho_t^f \\ &= \mathrm{E}_t y_{t+1}^h - \left(\frac{1}{\sigma}\right)\left[1 - (2-\gamma)\gamma + (2-\gamma)\gamma\sigma a\right]\rho_t^h - \left(\frac{1}{\sigma}\right)\left[(1-\gamma)\gamma - (2-\gamma)\gamma\sigma a + \gamma\right]\rho_t^f \\ &= \mathrm{E}_t y_{t+1}^h - \left(\frac{1}{\sigma}\right)\left[1 + (2-\gamma)\gamma(\sigma a - 1)\right]\rho_t^h - \left(\frac{1}{\sigma}\right)\left[(1-\gamma)\gamma - (2-\gamma)\gamma\sigma a + \gamma\right]\rho_t^f \\ &= \mathrm{E}_t y_{t+1}^h - \left(\frac{1+w}{\sigma}\right)\rho_t^h - \left(\frac{1}{\sigma}\right)\left[(1-\gamma)\gamma - (2-\gamma)\gamma\sigma a + \gamma\right]\rho_t^f \end{aligned}$$

where

$$w = (2-\gamma)\gamma(\sigma a - 1).$$

This Euler expression can further be written as

$$\begin{aligned} y_t^h &= \mathrm{E}_t y_{t+1}^h - \left(\frac{1+w}{\sigma}\right)\rho_t^h - \left(\frac{1}{\sigma}\right)\left[(2-\gamma)\gamma - (2-\gamma)\gamma\sigma a\right]\rho_t^f \\ &= \mathrm{E}_t y_{t+1}^h - \left(\frac{1+w}{\sigma}\right)\rho_t^h + \left(\frac{1}{\sigma}\right)(2-\gamma)\gamma(\sigma a - 1)\rho_t^f \\ &= \mathrm{E}_t y_{t+1}^h - \left(\frac{1+w}{\sigma}\right)\rho_t^h + \left(\frac{w}{\sigma}\right)\rho_t^f. \end{aligned}$$

so

$$y_t = \mathrm{E}_t y_{t+1} - \left(\frac{1+w}{\sigma}\right)\left[\rho_t^h - \left(\frac{w}{1+w}\right)\rho_t^f\right]. \tag{15}$$

Equation (15) implies

$$\rho_t^h = \left(\frac{\sigma}{1+w}\right)\left(\mathrm{E}_t y_{t+1}^h - y_t^h\right) + \frac{w}{1+w}\rho_t^f. \tag{16}$$

But from (14), $\rho_t^h = \rho_t^f + (\mathrm{E}_t\delta_{t+1} - \delta_t)$, so (16) becomes

$$\rho_t^h = \rho_t^f + (\mathrm{E}_t\delta_{t+1} - \delta_t) = \left(\frac{\sigma}{1+w}\right)\left(\mathrm{E}_t y_{t+1}^h - y_t^h\right) + \frac{w}{1+w}\rho_t^f,$$

or

$$\mathrm{E}_t\delta_{t+1} - \delta_t = \left(\frac{\sigma}{1+w}\right)\left(\mathrm{E}_t y_{t+1}^h - y_t^h\right) - \frac{1}{1+w}\rho_t^f.$$

Since $\rho_t^f = \sigma\left(\mathrm{E}_t y_{t+1}^f - y_t^f\right)$, this last equation can be written as

$$\mathrm{E}_t\Delta\delta_{t+1} = \left(\frac{\sigma}{1+w}\right)\left(\mathrm{E}_t\Delta y_{t+1}^h - \mathrm{E}_t\Delta y_{t+1}^f\right). \tag{17}$$

From (13),

$$y_t^h = (1-\gamma)(c_t - y_t^h) + (1-\gamma)y_t^h + (2-\gamma)\gamma a\delta_t + \gamma y_t^f.$$

Then,

$$\begin{aligned}
(1-\gamma)(c_t - y_t^h) &= \gamma y_t^h - (2-\gamma)\gamma a\delta_t - \gamma y_t^f \\
&= \gamma(y_t^h - y_t^f) - \gamma\left(\mathrm{E}_t y_{t+1}^h - \mathrm{E}_t y_{t+1}^f\right) + \gamma\left(\mathrm{E}_t y_{t+1}^h - \mathrm{E}_t y_{t+1}^f\right) - (2-\gamma)\gamma a\delta_t \\
&= -\gamma\left(\mathrm{E}_t\Delta y_{t+1}^h - \mathrm{E}_t\Delta y_{t+1}^f\right) + \gamma\left(\mathrm{E}_t y_{t+1}^h - \mathrm{E}_t y_{t+1}^f\right) - (2-\gamma)\gamma a\delta_t \\
&= \left(\frac{1}{\sigma}\right)\left[\gamma(1+w) - (2-\gamma)\gamma\sigma a\right]\delta_t - \frac{\gamma(1+w)}{\sigma}\mathrm{E}_t\delta_{t+1} + \gamma\left(\mathrm{E}_t y_{t+1}^h - \mathrm{E}_t y_{t+1}^f\right) \\
&= \left(\frac{1}{\sigma}\right)\left[\gamma(1+w) - (2-\gamma)\gamma\sigma a\right]\delta_t - \frac{\gamma(1+w)}{\sigma}\mathrm{E}_t\delta_{t+1} + \gamma\left(\mathrm{E}_t y_{t+1}^h - \mathrm{E}_t y_{t+1}^f\right).
\end{aligned}$$

But

$$\begin{aligned}
\gamma + \gamma w - (2-\gamma)\gamma\sigma a &= \gamma + \gamma(2-\gamma)\gamma(\sigma a - 1) - (2-\gamma)\gamma\sigma a \\
&= \gamma + (2-\gamma)\gamma^2\sigma a - \gamma^2(2-\gamma) - (2-\gamma)\gamma\sigma a \\
&= \gamma - \gamma^2(2-\gamma) + (\gamma-1)(2-\gamma)\gamma\sigma a \\
&= \gamma(1 - 2\gamma + \gamma^2) + (\gamma-1)(2-\gamma)\gamma\sigma a \\
&= (1-\gamma)\gamma\left[1 - \gamma - (2-\gamma)\sigma a\right] \\
&= -(1-\gamma)\left[(2-\gamma)\gamma\sigma a - \gamma + \gamma^2\right] \\
&= -(1-\gamma)\left[(2-\gamma)\gamma\sigma a - (1-\gamma)\gamma - \gamma + \gamma\right] \\
&= -(1-\gamma)\left[(2-\gamma)\gamma\sigma a - (2-\gamma)\gamma + \gamma\right] \\
&= -(1-\gamma)\left[(2-\gamma)\gamma(\sigma a - 1) + \gamma\right] \\
&= -(1-\gamma)\left[w + \gamma\right]
\end{aligned}$$

so we end up with

$$(1-\gamma)(c_t - y_t^h) = -\left(\frac{1}{\sigma}\right)(1-\gamma)(w+\gamma)\delta_t - \frac{\gamma(1+w)}{\sigma}\mathrm{E}_t\delta_{t+1} + \gamma\left(\mathrm{E}_t y_{t+1}^h - \mathrm{E}_t y_{t+1}^f\right)$$

or

$$y_t^h = c_t + \left(\frac{w+\gamma}{\sigma}\right)\delta_t + \frac{\gamma(1+w)}{\sigma}\mathrm{E}_t\delta_{t+1} - \gamma\left(\mathrm{E}_t y_{t+1}^h - \mathrm{E}_t y_{t+1}^f\right). \tag{18}$$

Section 6.5.2: The Flexible-Price Equilibrium

Assume that the productivity and wage markup disturbances are mean zero, white noise disturbances. Let z_t^o denote the flexible-price equilibrium value of a variable z_t (still expressed as percentage deviations around the steady state).

From the marginal cost condition and the production function with flexible prices,

$$\eta(y_t^o - \varepsilon_t) + \sigma c_t^o = w_t^o - p_t^c = w_t - p_t^h - \gamma\delta_t = \varepsilon_t - \gamma\delta_t^o,$$

or

$$\eta y_t^o + \sigma c_t^o = (1+\eta)\varepsilon_t - \gamma\delta_t^o.$$

Rewrite this as

$$(\eta + \sigma) y_t^o + \sigma(c_t^o - y_t^o) = (1+\eta)\varepsilon_t - \gamma\delta_t^o.$$

Using (18), this becomes

$$(\eta + \sigma) y_t^o - (w + \gamma)\delta_t^o = (1+\eta)\varepsilon_t - \gamma\delta_t^o,$$

or

$$(\eta + \sigma) y_t^o - w\delta_t^o = (\eta + \sigma) y_t^o - \left(\frac{\sigma w}{1+w}\right)\left(y_t^o - y_t^f\right) = (1+\eta)\varepsilon_t,$$

which becomes

$$(1+w)(\eta + \sigma) y_t^o - \sigma w y_t^o + \sigma w y_t^f = (1+w)(1+\eta)\varepsilon_t$$

or

$$\left[(1+w)\eta + \sigma\right] y_t^o + \sigma w y_t^f = (1+w)(1+\eta)\varepsilon_t \tag{19}$$

so

$$y_t^o = \left[\frac{(1+w)(1+\eta)}{(1+w)\eta + \sigma}\right]\varepsilon_t - \left[\frac{\sigma w}{(1+w)\eta + \sigma}\right] y_t^f. \tag{20}$$

Then, the flexible-price equilibrium satisfies

$$y_t^o = c_t^o + \left(\frac{w + \gamma}{\sigma}\right)\delta_t^o. \tag{21}$$

$$y_t^o = \left[\frac{(1+w)(1+\eta)}{(1+w)\eta + \sigma}\right]\varepsilon_t - \left[\frac{\sigma w}{(1+w)\eta + \sigma}\right] y_t^f. \tag{22}$$

$$\rho_t^o = r_t - \mathrm{E}_t\pi_{t+1}^h = \rho_t^f - \delta_t^o \tag{23}$$

$$\delta_t^o = \left(\frac{\sigma}{1+w}\right)\left(y_t^o - y_t^f\right). \tag{24}$$

Section 6.5.3: Deviations from the Flexible-Price Equilibrium
Define the output gap x_t as

$$x_t \equiv y_t - y_t^o.$$

Real marginal cost, given by (8), is equal to the gap between the real product wage and the marginal product of labor. When prices are sticky, the real wage can deviate from the marginal product of labor, but with flexible wages, the real consumption wage is still equal to the marginal rate of substitution between leisure and consumption. Thus,

$$w_t - p_t^h - \varepsilon_t = (\eta n_t + \sigma c_t + \mu_t^w + \gamma \delta_t) - \varepsilon_t.$$

From the production function and (18),

$$\begin{aligned} \eta n_t + \sigma c_t + \mu_t^w + \gamma \delta_t &= \eta(y_t^h - \varepsilon_t) + \sigma \left(y_t^h - \left(\frac{w+\gamma}{\sigma} \right) \delta_t \right) + \mu_t^w + \gamma \delta_t \\ &= (\eta + \sigma) y_t^h - \eta \varepsilon_t - w \delta_t + \mu_t^w. \end{aligned}$$

In terms of deviations from the flex-price equlbrium,

$$mc_t = (\eta + \sigma) x_t - w(\delta_t - \delta_t^o) + \mu_t^w.$$

From (17), $\delta_t - \delta_t^o = (\sigma/(1+w)) y_t^o$, so

$$mc_t = \left[\eta + \sigma - \left(\frac{\sigma w}{1+w} \right) \right] x_t + \mu_t^w.$$

Hence, using (9) implies that domestic inflation is[1]

$$\pi_t^h = \beta \mathrm{E}_t \pi_{t+1}^h + \kappa \left[\sigma + \eta - \left(\frac{\sigma w}{1+w} \right) \right] x_t + \kappa \mu_t^w. \tag{25}$$

From (15),

$$\begin{aligned} x_t &= \mathrm{E}_t x_{t+1} - \left(\frac{1+w}{\sigma} \right) \left[i_t - \mathrm{E}_t \pi_{t+1}^h - \left(\frac{w}{1+w} \right) \rho_t^f \right] + \mathrm{E}_t y_{t+1}^o - y_t^o \\ &= \mathrm{E}_t x_{t+1} - \left(\frac{1+w}{\sigma} \right) \left[i_t - \mathrm{E}_t \pi_{t+1}^h - \rho_t^o \right], \end{aligned} \tag{26}$$

where ρ_t^o is the equilibrium real interest rate under flexible prices, given by (24).

[1] Clarida, Galí, and Gertler (2002) assume that the stochastic wage markup μ_t^w does not affect the flexible–price equilibrium, so it appears in as a disturbance in teh inflation equation. If μ^w is viewed as a taste disturbance, it should be incorporated into the definition of the flexible-price equilibrium and, in this case, it would not enter the inflation equationindependently of the output gap variable.