Math 22
Midterm 1 Review Problems

Note: these problems are not a complete representation of all topics that may appear on midterm 1.

1. Let $\mathbf{a} = <1, 2, -1>$, $\mathbf{b} = <2, 3, -5>$, and $\mathbf{c} = <-3, 1, 4>$. Find the following:
   a. $\mathbf{a} \times \mathbf{b}$
   b. $\mathbf{b} \bullet \mathbf{c}$
   c. The area of the parallelogram spanned by $\mathbf{a}$ and $\mathbf{b}$.
   d. The volume of the parallelepiped spanned by $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c}$.
   e. $\text{comp}_{\mathbf{c}}\mathbf{a}$
   f. $\text{proj}_{\mathbf{b}}\mathbf{c}$
   g. A unit vector in the direction of $\mathbf{c}$.
   h. The angle between $\mathbf{a}$ and $\mathbf{b}$.
   i. The direction cosines of $\mathbf{c}$.
   j. The direction angles of $\mathbf{c}$.

2. Let $\mathbf{a} = <4, s^2, 3s>$ and $\mathbf{b} = <s, s, s+1>$. Determine all real numbers s such that:
   a. $\mathbf{a}$ is perpendicular to $\mathbf{b}$.
   b. $\mathbf{a}$ is parallel to $\mathbf{b}$.
   c. $\mathbf{a}$ is neither perpendicular nor parallel to $\mathbf{b}$.
   d. $\mathbf{a}$ is parallel to $<2, 9/2, 9/2>$

3. Find equations for:
   a. The sphere of radius 10 centered at $(2, 1, -1)$.
   b. The plane passing through the *z*-axis and the point $(1, 1, 20)$.
   c. The right circular cylinder parallel to the *y*-axis, generated by the circle of radius 5 in the *xz*-plane centered at $(5, 5)$.

4. Determine parametric and symmetric equations for the following lines.
   a. The *x*-axis.
   b. The line passing through $(2, 1, -3)$ and parallel to $<1, 0, 2>$.
   c. The intersection of the planes $x+y+z=-3$ and $2x+3y-z=5$.
   d. The tangent line at the point $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}, \frac{1-\sqrt{3}}{2}\right)$ to the curve C obtained by intersecting the plane $x+y+z=1$ with the cylinder $x^2+y^2=1$.

5. Set up a definite integral for the length L of the curve C described in problem 4d. Do not evaluate this integral.

6. Prove the following identities:
   a. $\mathbf{a} \bullet (\mathbf{b}+\mathbf{c}) = \mathbf{a} \bullet \mathbf{b} + \mathbf{a} \bullet \mathbf{c}$ (where $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c}$ are vectors in $\mathbf{R}^2$.)
   b. $\mathbf{a} \times (\mathbf{b}+\mathbf{c}) = \mathbf{a} \times \mathbf{b} + \mathbf{a} \times \mathbf{c}$ (where $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c}$ are vectors in $\mathbf{R}^3$.)
   c. $\frac{d}{dt}\left[f(t)\mathbf{r}(t)\right] = f'(t)\mathbf{r}(t) + f(t)\mathbf{r}'(t)$ (where $f(t)$ is a real function, and $\mathbf{r}(t)$ is a vector function.)

7. Identify each of the equations below as one of the following types of surface: Elliptic Cylinder, Parabolic Cylinder, Hyperbolic Cylinder, Sphere, Ellipsoid, Elliptic Paraboloid, Hyperbolic Paraboloid, Hyperboloic of 1 Sheet, or Hyperboloid of 2 Sheets. If the surface is centered at some point other than the origin, give the coordinates of that point. Sketch the surface.
   a. $(x-5)^2+(y-7)^2+(z+9)^2=36$
   b. $\dfrac{y^2}{3}+\dfrac{z^2}{4}=1$
   c. $\dfrac{y^2}{3}-\dfrac{z^2}{4}=1$
   d. $x=y^2+2z^2$
   e. $x=2z^2$
   f. $z=x^2-y^2$
   g. $\dfrac{x^2}{16}+\dfrac{y^2}{16}+\dfrac{z^2}{9}=1$
   h. $\dfrac{x^2}{16}+\dfrac{y^2}{16}-\dfrac{z^2}{9}=1$
   i. $\dfrac{x^2}{16}-\dfrac{y^2}{16}-\dfrac{z^2}{9}=1$
   j. $y-1=(x+1)^2-z^2$
   k. $-36x^2+16y^2-9z^2+216x+32y+36z-488=0$

8. Determine the velocity and acceleration vectors, as well as the speed, of the following vector functions at the specified point. Also write symmetric equations for the tangent lines these curves at the specified points.
   a. $\mathbf{r}(t)=<t^3,\ t^2,\ t>$ at $t=3$
   b. $\mathbf{r}(t)=<t\cos(t),\ t^2\sin(t)>$ at $t=\dfrac{2\pi}{3}$
   c. $\mathbf{r}(t)=<1+\sqrt{t},\ 1-\sqrt{t},\ 2e^t>$ at $(1,\ 0,\ 2e)$

9. For each of the curves given by the vector functions in problem 8, set up a definite integral for its length over the specified time interval. Do not evaluate these integrals.
   a. $1\le t\le 5$
   b. $0\le t\le 4\pi$
   c. $0\le t\le 1$

The following formulas will appear on the last page of the midterm 1 exam.

Some Indefinite Integrals

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C \quad (\text{for } n \neq -1)$$

$$\int x^{-1} dx = \ln|x| + C$$

$$\int e^x dx = e^x + C$$

$$\int a^x dx = \frac{a^x}{\ln a} + C$$

$$\int \cos x \, dx = \sin x + C$$

$$\int \sin x \, dx = -\cos x + C$$

$$\int \sec^2 x \, dx = \tan x + C$$

$$\int \csc^2 x \, dx = -\cot x + C$$

$$\int \sec x \tan x \, dx = \sec x + C$$

$$\int \csc x \cot x \, dx = -\csc x + C$$

$$\int \tan x \, dx = \ln|\sec x| + C$$

$$\int \cot x \, dx = -\ln|\csc x| + C$$

$$\int \frac{1}{1+x^2} dx = \tan^{-1} x + C$$

$$\int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1} x + C$$

Standard Angles