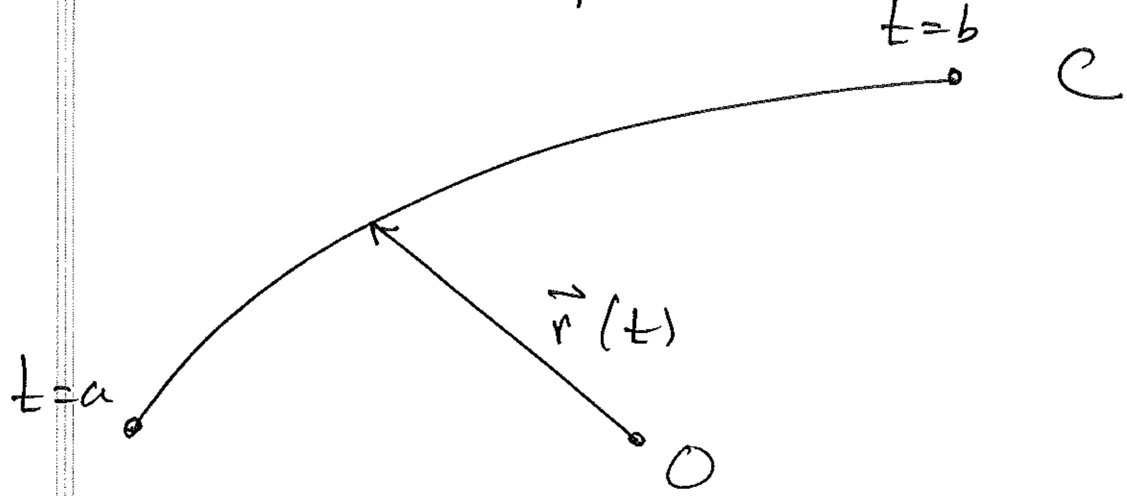
(13.4) VELOCITY AND ACCELERATIONS



RECALL THE LENGTH L OF $\mathcal{C}$ IS

$$L = \int_a^b |\vec{r}\,'(t)|\,dt \quad .$$

THIS FORMULA IMPLIES THAT $|\vec{r}\,'(t)|$ IS THE INSTANTANEOUS SPEED OF $\vec{r}(t)$ AS IT TRACES $\mathcal{C}$ FROM $\vec{r}(a)$ TO $\vec{r}(b)$.

THUS $\vec{r}\,'(t)$ IS THE <u>VELOCITY</u> OF A MOVING PARTICLE AT THE ARROW OF $\vec{r}(t)$, i.e.

(1) $\vec{r}\,'(t)$ IS TANGENT TO THE PARTICLE'S PATH

(2) $|\vec{r}\,'(t)|$ IS THE INSTANTANEOUS SPEED OF THE PARTICLE.

WE WRITE

$$\vec{v}(t) = \vec{r}\,'(t) = \frac{d}{dt}\left[\vec{r}(t)\right]$$

RMK

- SPEED IS A SCALAR QUANTITY, i.e. A REAL NUMBER, OR A REAL VALUED FUNCTION.

- VELOCITY IS A VECTOR QUANTITY, i.e. A VECTOR FUNCTION.

DEFN

THE ACCELERATION OF A PARTICLE MOVING ALONG $\vec{r}(t)$ IS THE VECTOR FUNCTION

$$\vec{a}(t) = \frac{d}{dt}\left[\vec{v}(t)\right] = \vec{v}\,'(t) = \vec{r}\,''(t).$$

EX FIND VELOCITY, ACCELERATION, & SPEED OF $\vec{r}(t) = \langle t, t^2, t^3 \rangle$ AT $(2, 4, 8)$.

NOTE: THE POINT $(2, 4, 8)$ CORRESPONDS TO $t = 2$.

$\vec{v}(t) = \langle 1, 2t, 3t^2 \rangle \quad \therefore \vec{v}(2) = \langle 1, 4, 12 \rangle$
$\vec{a}(t) = \langle 0, 2, 6t \rangle \quad \therefore \vec{a}(2) = \langle 0, 2, 12 \rangle$

speed $= |\vec{v}(2)| = \sqrt{161} = 12.68$

__EX.__
GIVEN $\vec{a}(t) = \langle t, \sqrt{t}, e^t + \frac{1}{t} \rangle$
AND $\vec{v}(1) = \langle \frac{1}{2}, \frac{2}{3}, e+1 \rangle$,
AND $\vec{r}(1) = \langle 1, 1, e \rangle$,

DETERMINE $\vec{v}(t)$ AND $\vec{r}(t)$.

ANS:

$$\vec{v}(t) = \langle \tfrac{1}{2}t^2, \tfrac{2}{3}t^{3/2}, e^t + \ln t + 1 \rangle$$

$$\vec{r}(t) = \langle \tfrac{1}{6}t^3 + \tfrac{5}{6}, \tfrac{4}{15}t^{5/2} + \tfrac{11}{15}, e^t + t\ln t \rangle$$

SKIP!
- TANGENT & NORMAL COMPONENTS OF ACCEL. —
- KEPLER'S LAWS —