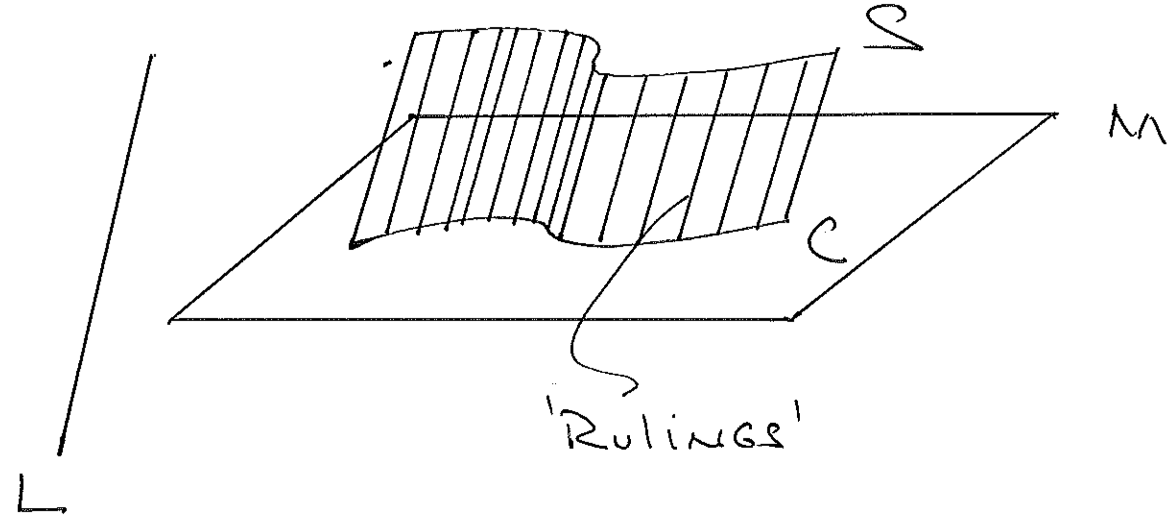
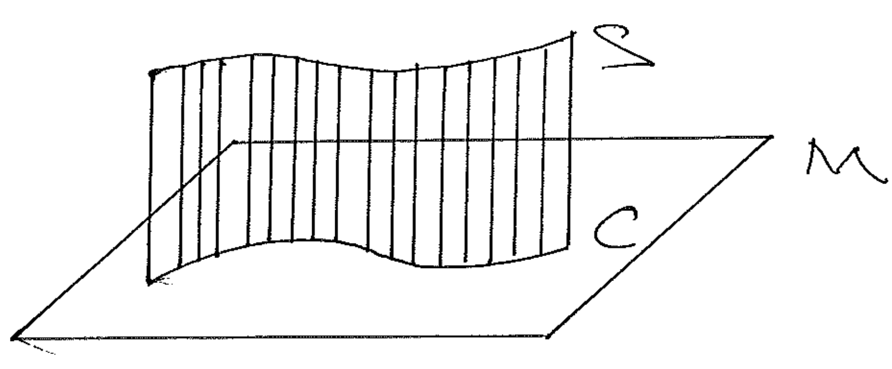
(12.6) Cylinders & Quadric Surfaces

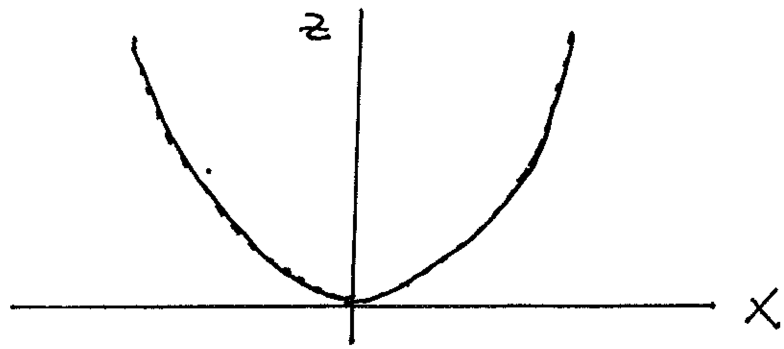
Defn.
Given a line L, and a plane curve C (lying in a plane M), the cylinder generated by L and C is the surface S consisting of all lines through C which are parallel to L.

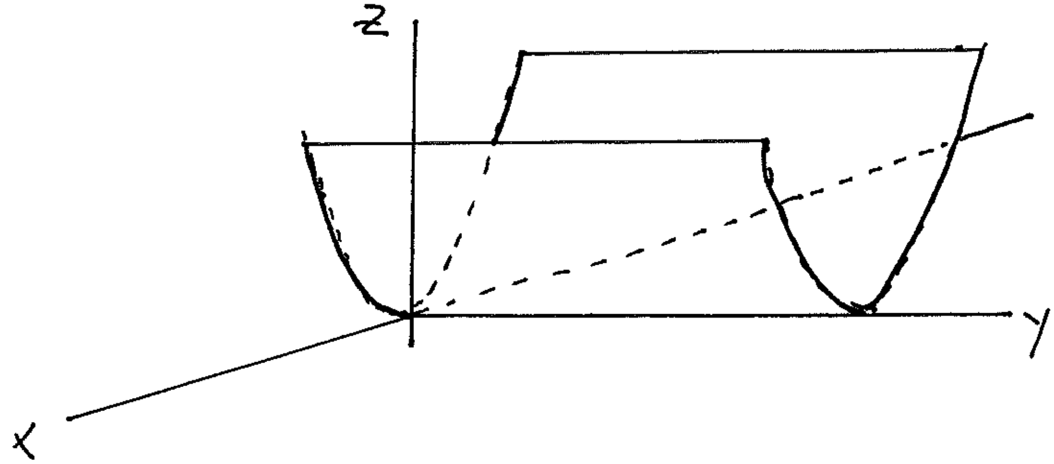


Note that L need not be perpendicular to M. If L is perpendicular to M, we call S a right cylinder

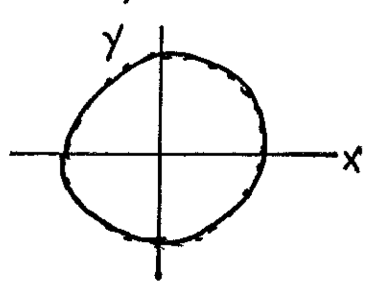


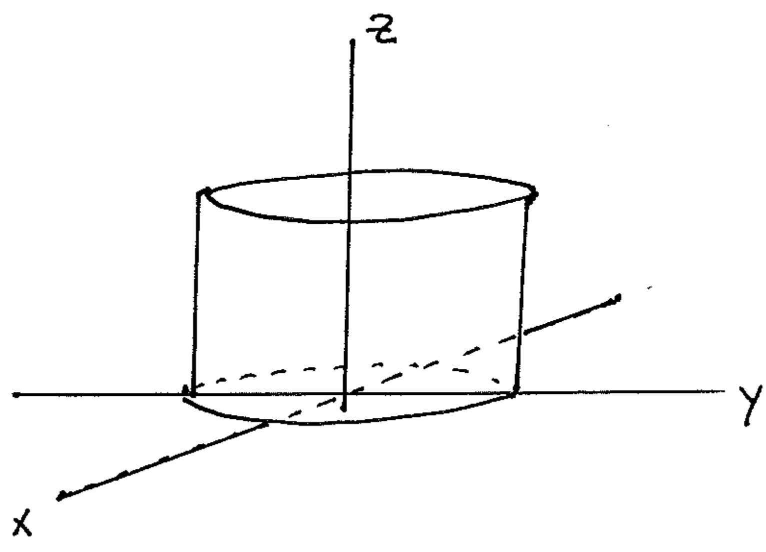
Ex. $z = x^2$: Right Parabolic Cylinder





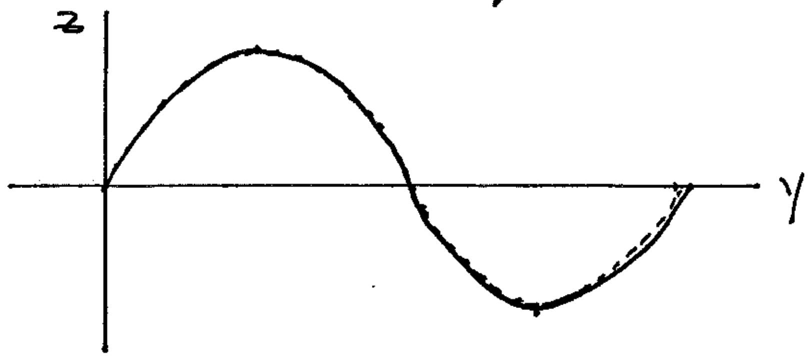
Ex. $x^2 + y^2 = 2$: Right Circular Cylinder

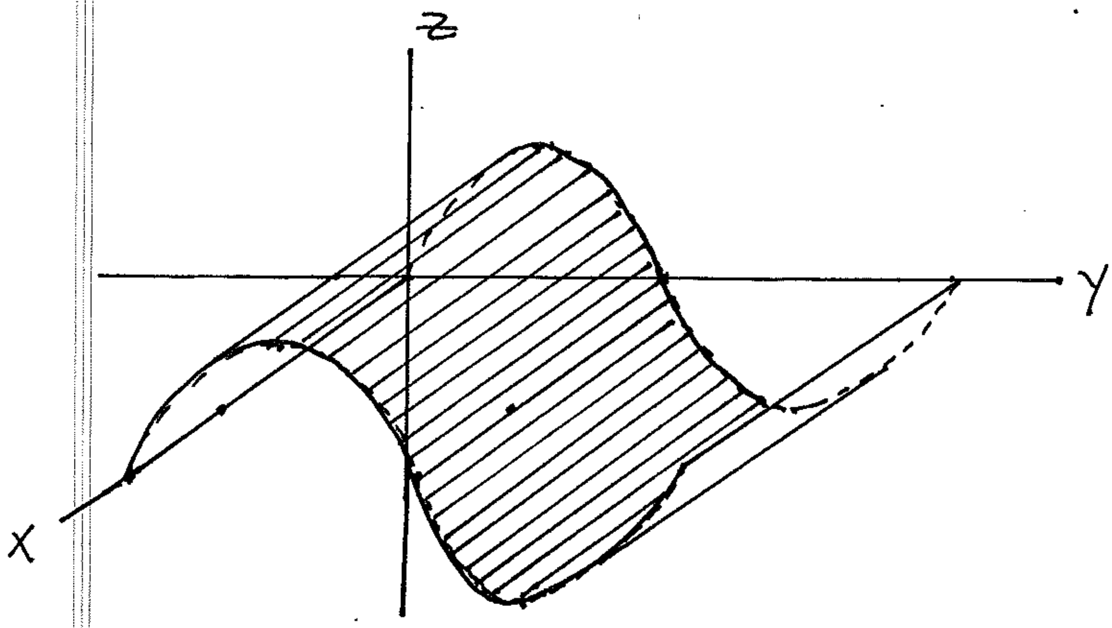




Ex. $z = \sin y \quad (0 \le y \le 2\pi)$

RIGHT SINUSOIDAL CYLINDER





DEFN.

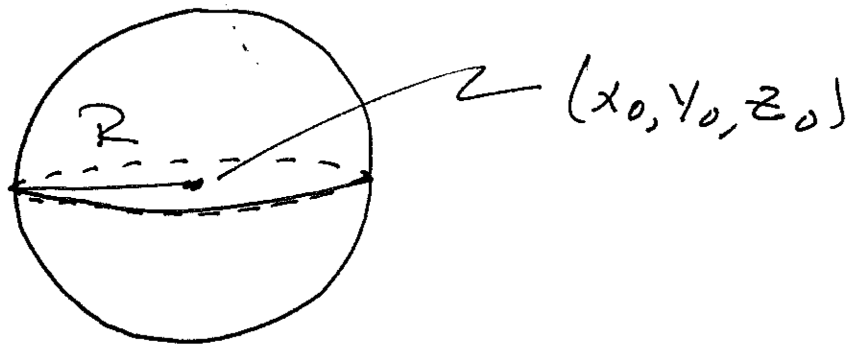
A QUADRIC SURFACE IS THE SET OF POINTS $(x, y, z) \in \mathbb{R}^3$ SATISFYING AN EQUATION OF THE FORM

$$Ax^2 + By^2 + Cz^2 + Dxy + Eyz + Fxz$$

$$+ Gx + Hy + Iz + J = 0$$

EX. SPHERE OF RADIUS R CENTERED AT (x_0, y_0, z_0).

$$(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2 = R^2$$

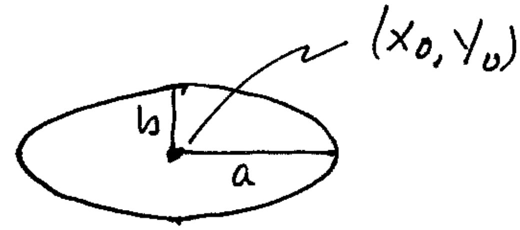


WE COULD WRITE THIS AS

$$\frac{(x-x_0)^2}{R^2} + \frac{(y-y_0)^2}{R^2} + \frac{(z-z_0)^2}{R^2} = 1$$

RECALL THE EQUATION OF AN ELLIPSE IN THE xy-PLANE IS

$$\frac{(x-x_0)^2}{a^2} + \frac{(y-y_0)^2}{b^2} = 1$$

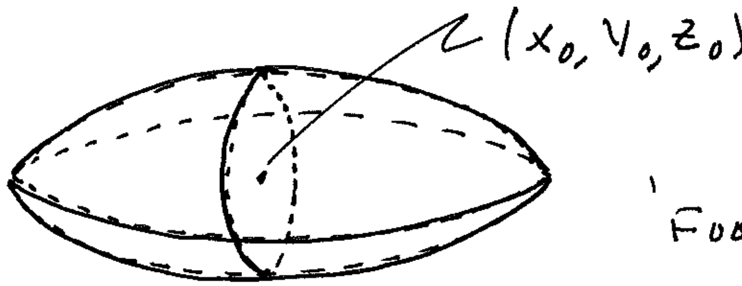


IF $a > b$ WE CALL a THE SEMI-MAJOR AXIS, AND b THE SEMI-MINOR AXIS, RESPECTIVELY.

Ex. ELLIPSOID

$$\frac{(x-x_0)^2}{a^2}+\frac{(y-y_0)^2}{b^2}+\frac{(z-z_0)^2}{c^2}=1$$

OBSERVE THAT THE INTERSECTIONS WITH THE PLANES $x=x_0$, $y=y_0$, AND $z=z_0$ ARE ELLIPSES.



'FOOTBALL'

THESE INTERSECTION CURVES ARE CALLED TRACES OR CROSS SECTIONS.

Ex. PARABOLOID

$$z = x^2+y^2$$

TRACES:

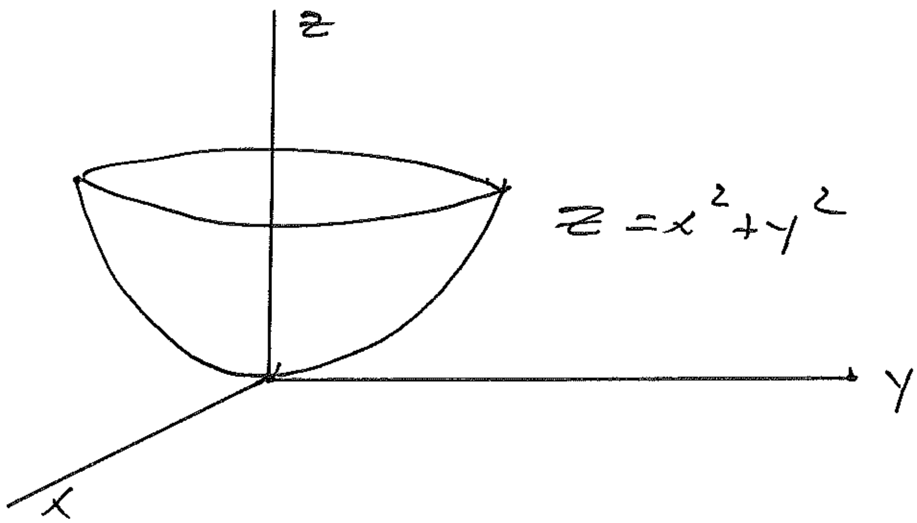
$z = z_0 < 0$: EMPTY

$z = 0$: $(0,0)$

$z = z_0 > 0$: CIRCLE $x^2+y^2=(\sqrt{z_0})^2$

$y = 0$: PARABOLA $z=x^2$

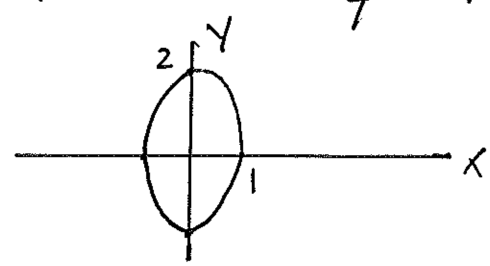
$x = 0$: PARABOLA $z=y^2$

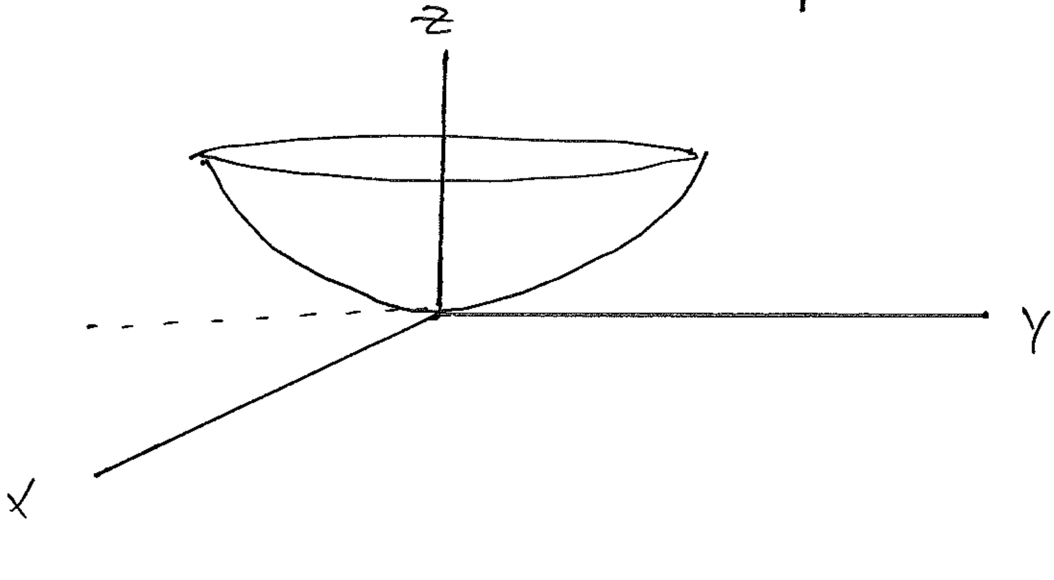


<u>EX.</u> ELLIPTIC PARABOLOID

$$z = x^2 + \frac{y^2}{4}$$

TRACE : $z = 1$: ELLIPSE $x^2 + \frac{y^2}{4} = 1$

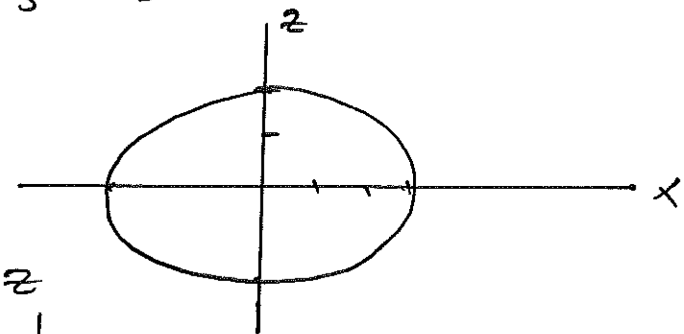


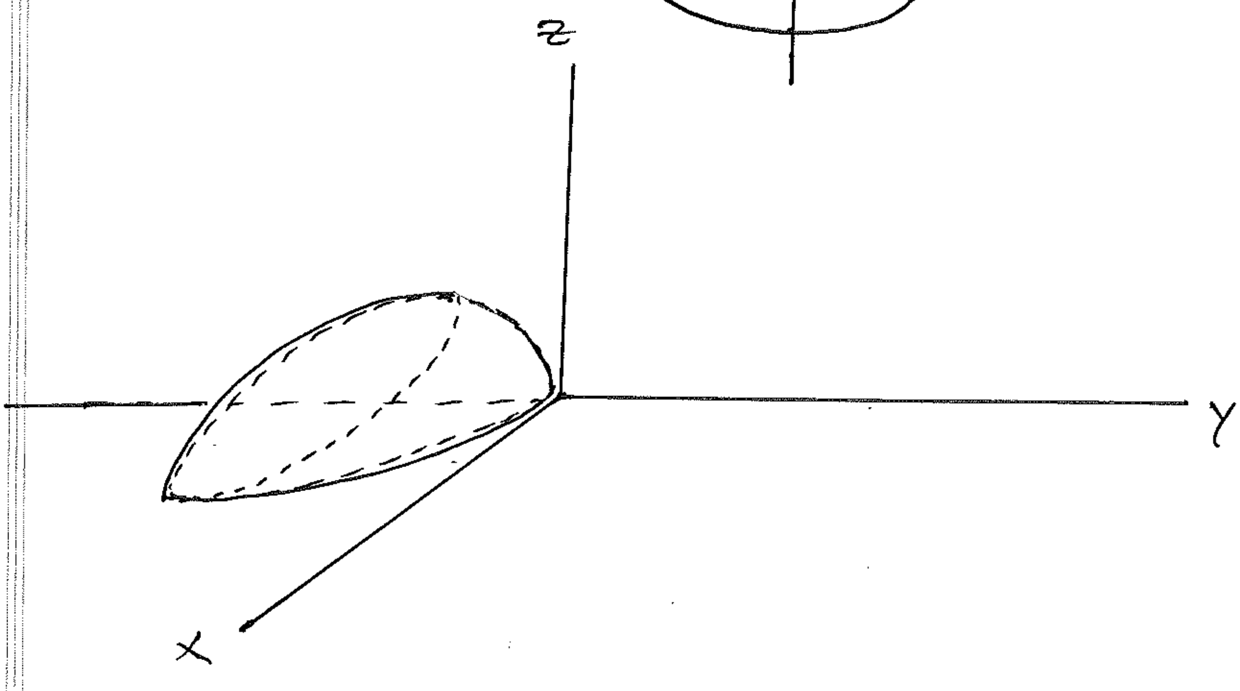


EX. ELLIPTIC PARABOLOID

$$y = -\frac{x^2}{9} - \frac{z^2}{4}$$

TRACE $y = -1$: $\frac{x^2}{3^2} + \frac{z^2}{2^2} = 1$ ELLIPSE

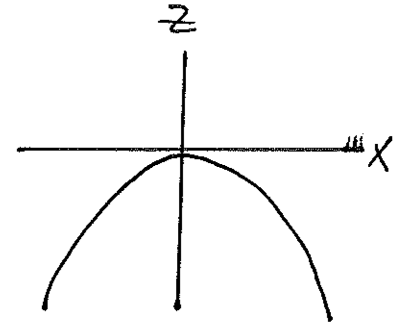




EX. HYPERBOLIC PARABOLOID

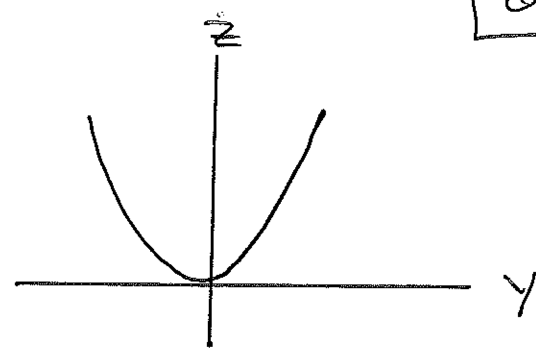
$$z = y^2 - x^2$$

TRACES : $y = 0 \rightarrow z = -x^2$
PARABOLA



$x = 0 \rightarrow z = y^2$

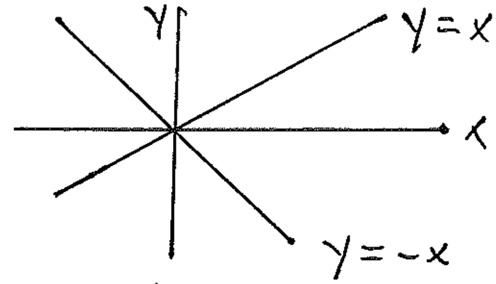
PARABOLA



$z = 0 \rightarrow x^2 = y^2$

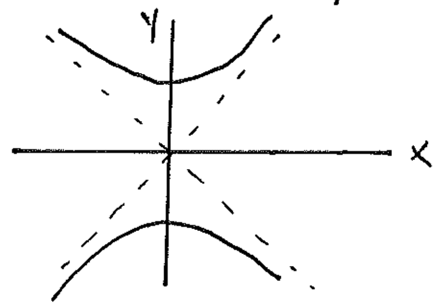
$y = \pm x$

TWO LINES



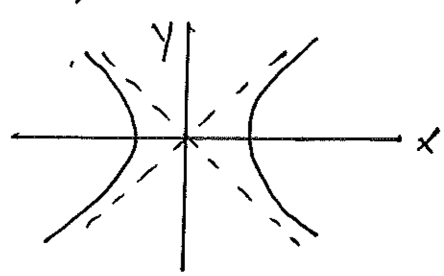
$z = 1 \rightarrow y^2 - x^2 = 1$

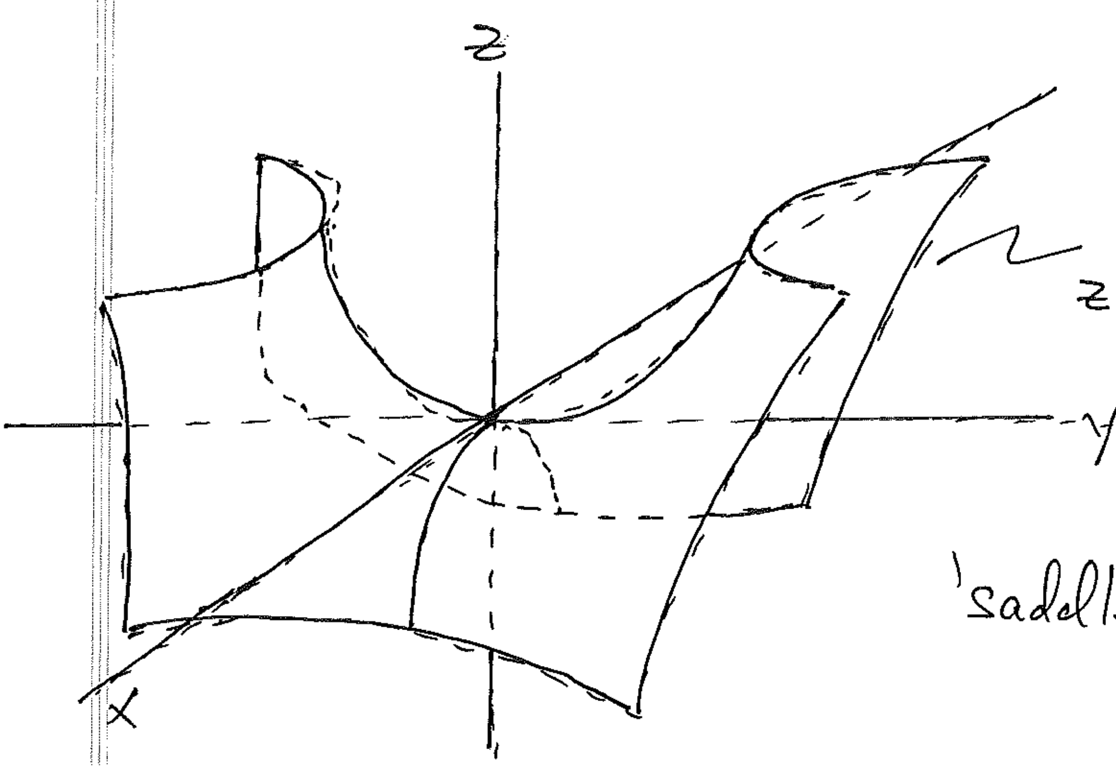
HYPERBOLA



$z = -1 \rightarrow x^2 - y^2 = 1$

HYPERBOLA





$z = y^2 - x^2$

'saddle'

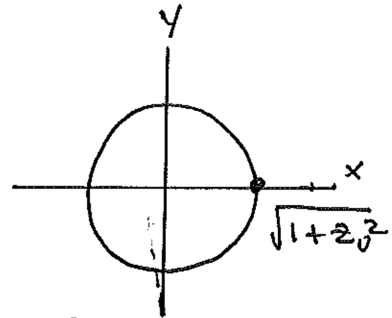
EX. HYPERBOLOID OF 1 SHEET

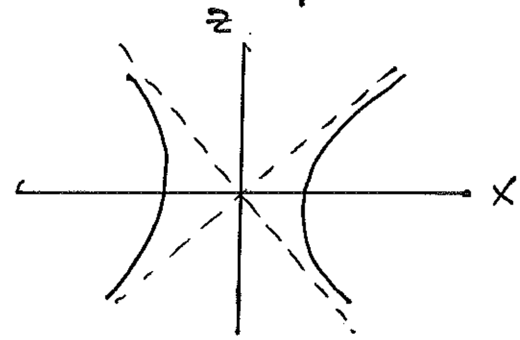
$$x^2+y^2-z^2=1$$

TRACES:

$z = z_0 \longrightarrow x^2+y^2 = 1+z_0^2$

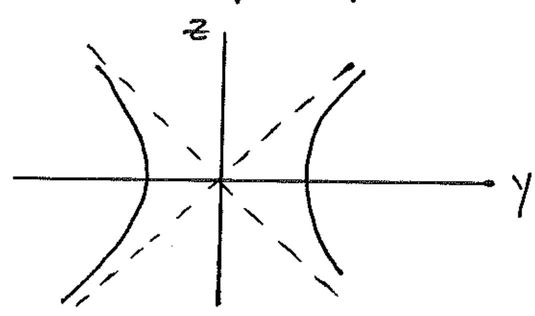
CIRCLE



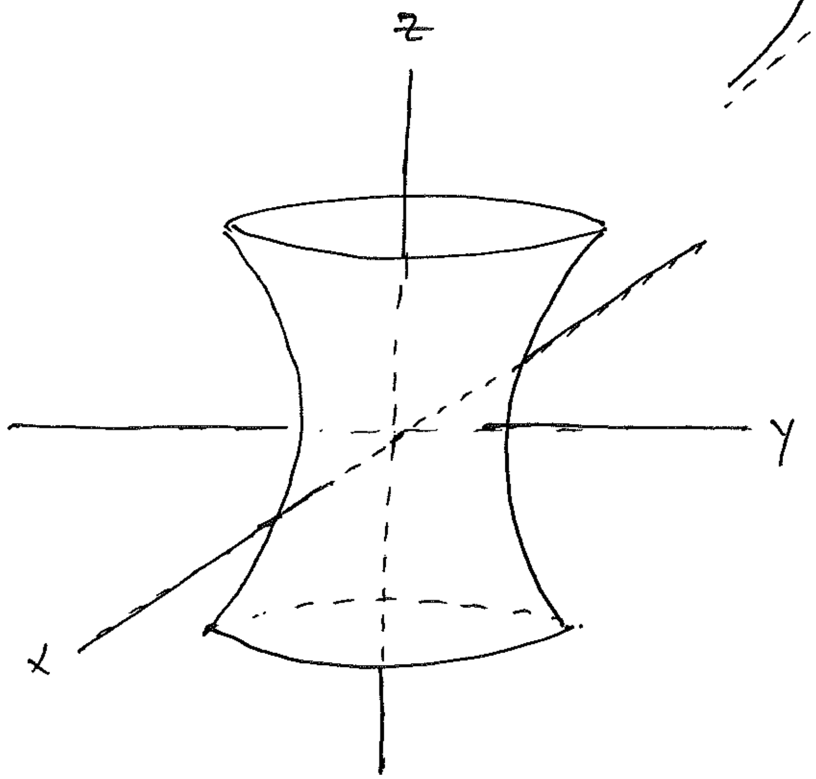


$y=0 \longrightarrow x^2-z^2=1$

HYPERBOLA



$x=0 \longrightarrow y^2-z^2=1$



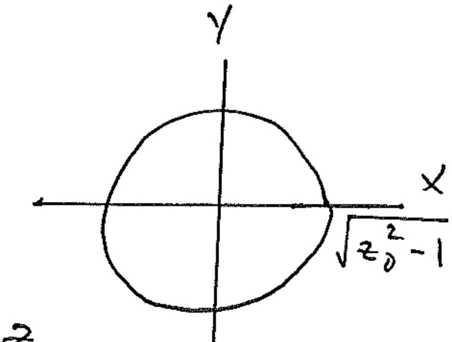
Ex. HYPERBOLOID OF 2 SHEETS

$$-x^2 - y^2 + z^2 = 1$$

TRACES:

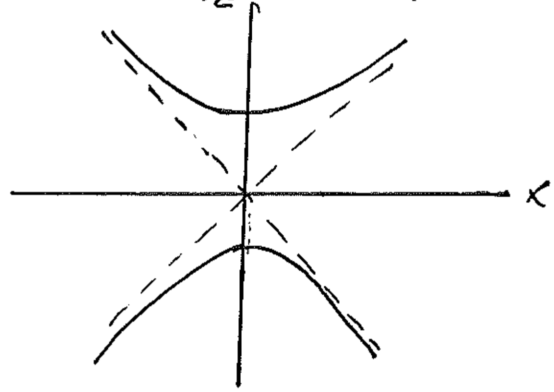
$z = z_0 \longrightarrow x^2 + y^2 = z_0^2 - 1$

- CIRCLE IF $z_0^2 \geq 1$
- EMPTY IF $z_0^2 < 1$



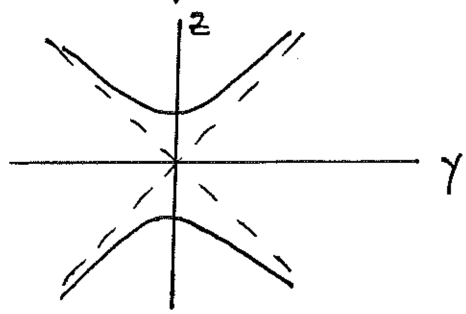
$y = 0 \longrightarrow z^2 - x^2 = 1$

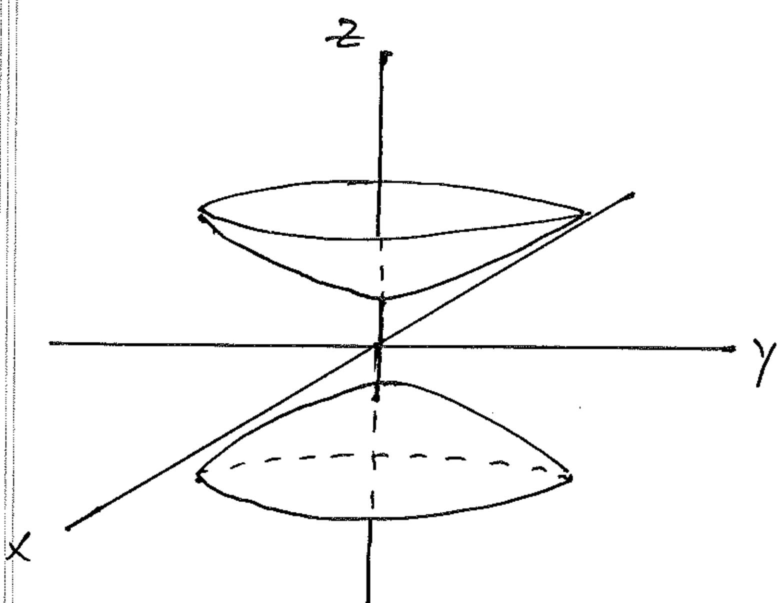
HYPERBOLA



$x = 0 \longrightarrow z^2 - y^2 = 1$

HYPERBOLA





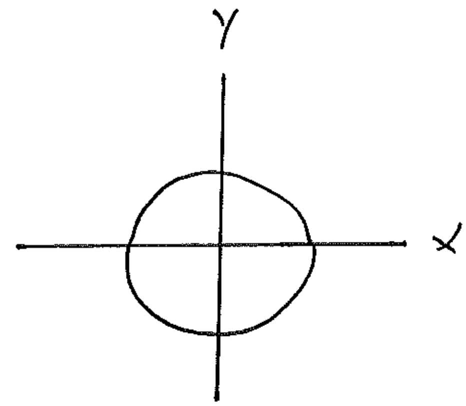
Ex. Cone

$$x^2 + y^2 - z^2 = 0$$

Traces

$z = z_0 \rightarrow x^2 + y^2 = z_0^2$

circle



$y = 0 \rightarrow z^2 = x^2 \rightarrow z = \pm x$ lines

or

$x = 0 \rightarrow z^2 = y^2 \rightarrow z = \pm y$ lines