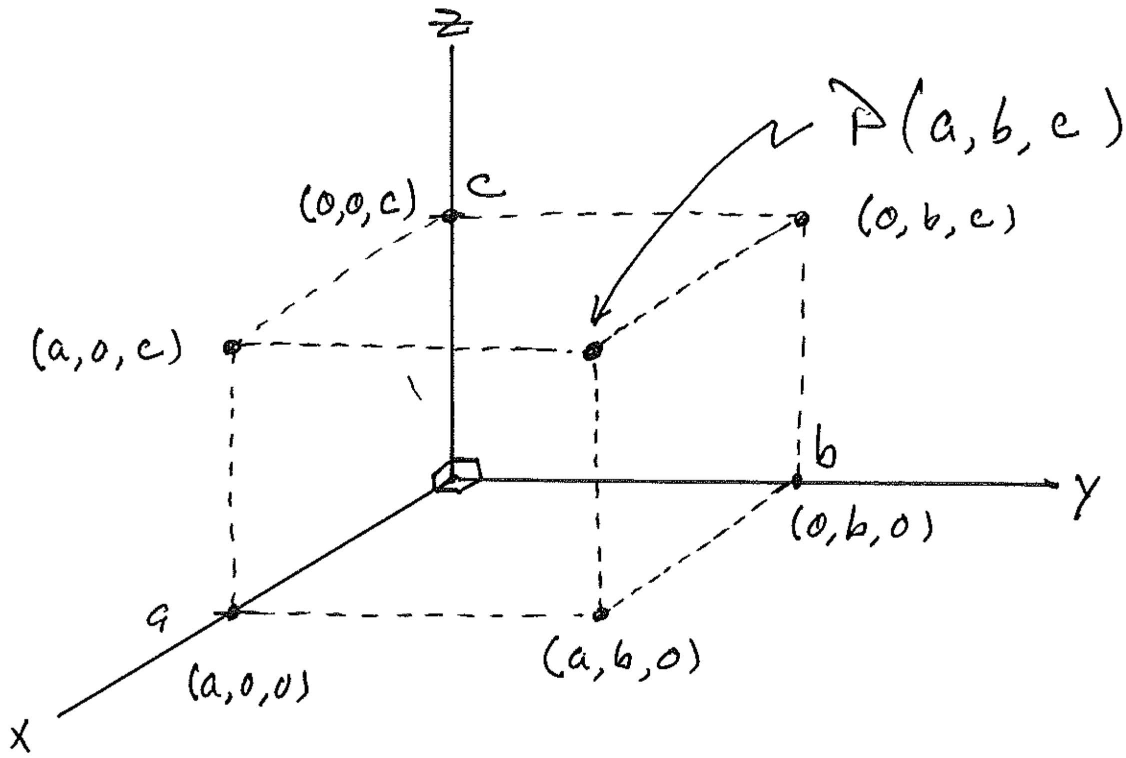
MATH 22
Calculus of Several Variables

(12.1) 3-D Coordinates

To locate a point in space we need 3 numbers: (a,b,c)



(a,b,c) are the <u>coordinates</u> of the point P

The projections of P onto:

xy - Plane : $(a,b,0)$
xz - Plane : $(a,0,c)$
yz - Plane : $(0,b,c)$
x - Axis : $(a,0,0)$
y - Axis : $(0,b,0)$
z - Axis : $(0,0,c)$

3-Dimensional space is sometimes Refered to as $\mathbb{R}^3$.

THE 3 COORDINATE PLANES DIVIDE $\mathbb{R}^3$ INTO 8 <u>OCTANTS</u>. THE 1^{ST} OCTANT IS CHARACTERIZED BY ALL COORDINATES BEING POSITIVE: $x>0,\ y>0,\ z>0$.

THE OCTANTS CAN BE SYMBOLIZED BY THE ORDERED TRIPLES

$$(+,+,+)$$
$$(+,+,-)$$
$$(+,-,+)$$
$$(+,-,1)$$
$$(-,+,+)$$
$$(-,+,-)$$
$$(-,-,+)$$
$$(-,-,-)$$

OBSERVE THAT THE xy-PLANE IS CHARACTERIZED BY THE EQUATION

$$z=0$$

i.e. THIS PLANE IS THE SET OF POINTS GIVEN BY

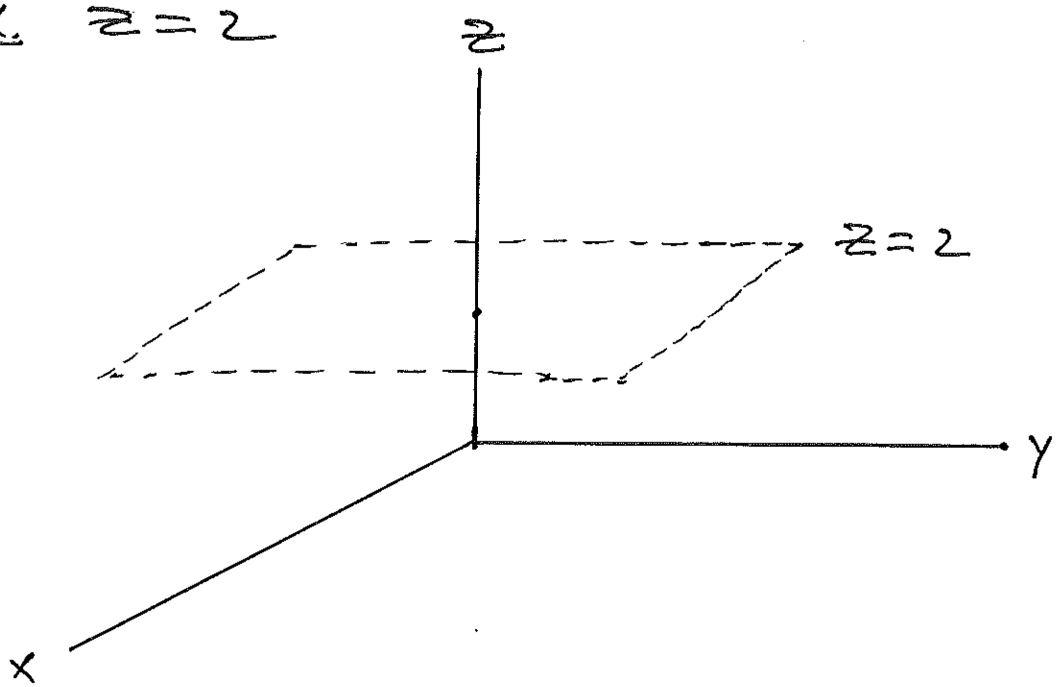
$$\{(x, y, 0) \mid x \in \mathbb{R}, y \in \mathbb{R}\}$$

IN GENERAL, THE SOLUTION SET OF AN EQUATION OF THE FORM

$$z = \text{const}$$

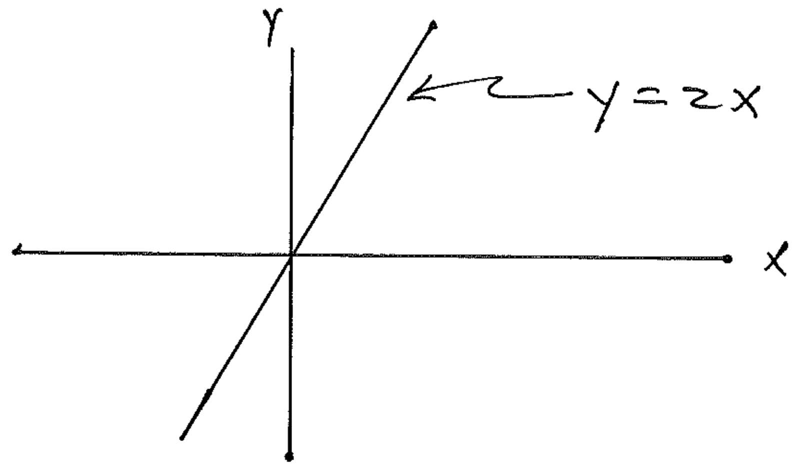
is A PLANE PARALLEL TO THE xy-PLANE.

EX. $z = 2$



Similarly EQUATIONS $y = \text{Const}$ & $x = \text{Const}$. REPRESENT PLANES PARALLEL TO THE xz-PLANE & yz-PLANE RESPECTIVELY.

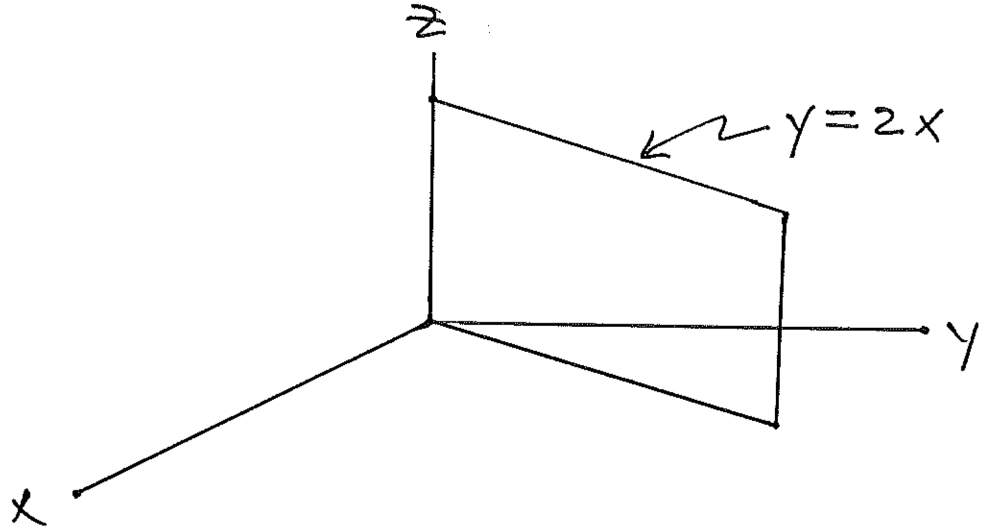
Ex Consider the equation $y = 2x$. In the xy plane this would represent a line



In $\mathbb{R}^3$ the solution set of this equation would be

$$\{(x, 2x, z) \mid x \in \mathbb{R}, z \in \mathbb{R}\}$$

which is the plane containing the z-axis and the above line:

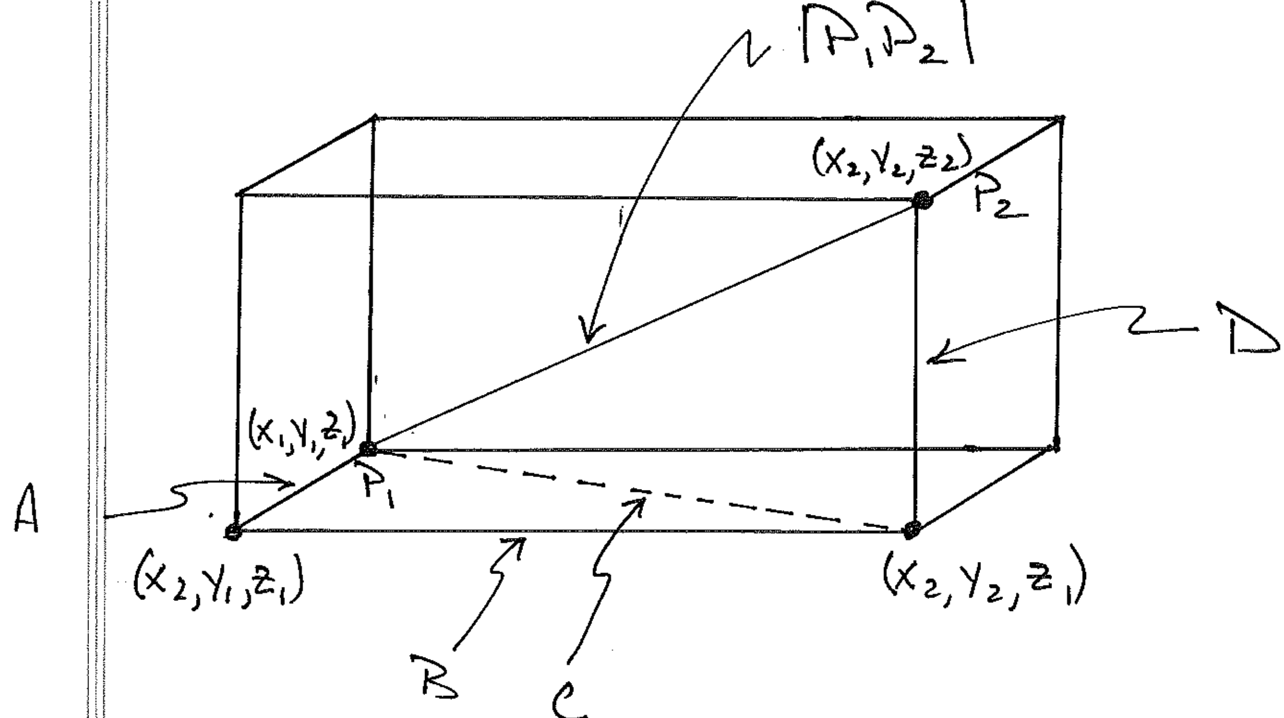


CONSIDER TWO POINTS P_1 & P_2 WITH COORDINATES (x_1, y_1, z_1) AND (x_2, y_2, z_2) RESP.

WE WISH TO FIND THE DISTANCE:

$$|P_1P_2| = \text{distance from } P_1 \text{ to } P_2.$$

CONSIDER THE RECTANGULAR BOX:



$A = |x_2 - x_1|$
$B = |y_2 - y_1|$
$C = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$ $\quad D = |z_2 - z_1|$

$$\therefore \boxed{|P_1P_2| = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}}$$

A sphere with center $P_0(x_0, y_0, z_0)$ AND RADIUS $r > 0$ consists of all POINTS $P(x, y, z)$ whose Distance from P_0 is r:

$$|PP_0| = r$$

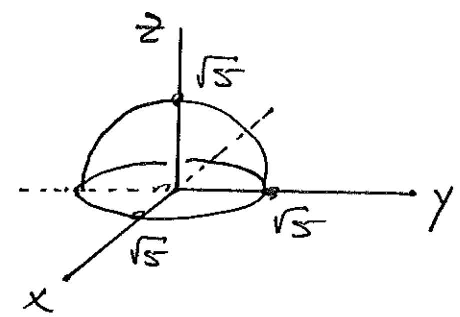
$$\therefore \quad |PP_0|^2 = r^2$$

$$\therefore \quad \boxed{(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2 = r^2}$$

THIS IS THE EQUATION OF A SPHERE. IF P_0 IS THE ORIGIN $(0,0,0)$ WE HAVE THE EQUATION

$$\boxed{x^2 + y^2 + z^2 = r^2}$$

Ex. $x^2 + y^2 + z^2 = 5$ is THE sphere of radius $\sqrt{5}$ CENTERED AT THE ORIGIN $(0,0,0)$:



A SOLID SPHERE (ALSO CALLED A BALL) INCLUDES THE POINTS INTERIOR TO THE SPHERE

EX. $x^2+y^2+z^2 \leq 5$ (CLOSED BALL)

EX. $x^2+y^2+z^2 < 5$ (OPEN BALL)

EX. $2 \leq (x-3)^2+(y+1)^2+(z-1)^2 < 10$
THIS IS THE SOLID REGION LYING BETWEEN THE SPHERES OF RADII $\sqrt{2}$ AND $\sqrt{10}$ BOTH CENTERED AT $(3,-1,1)$. THE REGION INCLUDES POINTS ON THE INNER SPHERE BUT NOT THE OUTER SPHERE.

EX. $y \geq 2x$

REPRESENTS THE 'HALF-SPACE' IN 'BACK' OF THE PLANE $y=2x$.