

Math 11B

Midterm 2 Review Problems

Note: these problems are not a representation of all topics that may appear on midterm 2.

1. Evaluate the following indefinite integrals.

a. $\int \frac{2}{3x+4} dx$

b. $\int \frac{2x}{3x+4} dx$

c. $\int xe^{x^2} dx$

d. $\int x^3 e^{x^2} dx$

e. $\int \frac{1}{x^2-9} dx$

f. $\int \frac{1}{x^2+9} dx$

g. $\int \frac{10x+6}{(x+1)^2} dx$

h. $\int \frac{x^4 + 3x^3 - 2x^2 + x + 1}{x^2 + 2x + 1} dx$

i. $\int e^x \sin x dx$

2. Evaluate the following definite integrals.

a. $\int_1^2 x^2 \ln(x^2) dx$

b. $\int_e^{e^2} \frac{1}{x(\ln x)^2} dx$

c. $\int_0^3 x^2 (x^3 + 100)^{1/3} dx$

d. $\int_0^\pi e^x \sin x dx$

e. $\int_0^{\pi/4} \tan x \sec^2 x e^{\tan x} dx$

f. $\int_0^\infty \frac{1}{x^2+9} dx$

g. $\int_0^1 \frac{1}{x^{2/3}} dx$

h. $\int_1^3 (x-2)^{-2/3} dx$

3. Determine the partial fraction decomposition of the following rational functions. (Do not integrate.)

a. $\frac{x^2 + 5}{(x-1)(x+2)^2}$

b. $\frac{x^2 + x + 1}{(x+1)(x^2 + 2)}$

4. Determine the *form* of a partial fraction decomposition of the following rational functions. (Do not determine the constants.)

a. $\frac{x^5 + x^4 + x^2 - x + 1}{(x-1)(2x+5)^2(3x-7)^3}$

b. $\frac{x^{10} + x^7 + x^3 + x + 3}{(x^2 + x + 1)(2x^2 + 5)^2(x^2 - 1)^3}$

5. For which p does $\int_0^1 \frac{1}{x^p} dx$ converge?

6. For which p does $\int_1^{\infty} \frac{1}{x^p} dx$ converge?

7. Determine the convergence or divergence of the following improper integrals. (Do not evaluate the integrals.)

a. $\int_1^{\infty} \frac{1}{\sqrt{x+\sqrt{x}}} dx$ (Hint: Show that $\frac{1}{\sqrt{2x}} \leq \frac{1}{\sqrt{x+\sqrt{x}}}$ for all $x \geq 1$.)

b. $\int_0^1 \frac{1}{\sqrt{x+\sqrt{x}}} dx$ (Hint: Show that $\frac{1}{\sqrt{x+\sqrt{x}}} < \frac{1}{\sqrt{x}}$ for all $0 < x \leq 1$.)

8. Solve the following initial value problem:
$$\begin{cases} \frac{dN}{dt} = te^t & t > 0 \\ N(0) = \frac{5}{2} \end{cases}$$

9. Determine the average value of the following functions on the indicated intervals.

a. $\ln x$ on $[1, e]$.

b. $\frac{1}{1+x^2}$ on $[0, 1]$

10. Determine the area of the following plane regions.

a. The region in the 1st quadrant bounded by $y = x^{-1/3}$, $y = 0$, $x = 0$, and $x = 1$.

b. The region in the 3rd quadrant bounded by $y = e^x$, $y = 0$, and $x = 0$.

c. The region in the 4th quadrant bounded by $y = \ln x$, $y = 0$, and $x = 0$.

Some Indefinite Integrals

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C \quad (\text{for } n \neq -1)$$

$$\int x^{-1} dx = \ln |x| + C$$

$$\int e^x dx = e^x + C$$

$$\int a^x dx = \frac{a^x}{\ln a} + C$$

$$\int \cos x dx = \sin x + C$$

$$\int \sin x dx = -\cos x + C$$

$$\int \sec^2 x dx = \tan x + C$$

$$\int \csc^2 x dx = -\cot x + C$$

$$\int \sec x \tan x dx = \sec x + C$$

$$\int \csc x \cot x dx = -\csc x + C$$

$$\int \tan x dx = \ln |\sec x| + C$$

$$\int \cot x dx = -\ln |\csc x| + C$$

$$\int \frac{1}{1+x^2} dx = \tan^{-1} x + C$$

$$\int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1} x + C$$

Standard Angles

