Math 11B
Midterm 1 Review Problems

Note: these problems are not a representation of all topics that may appear on midterm 2.

1. Solve the following initial value problems
   a. $\frac{dP}{ds} = 2s^2 + 3s - 1$, $P(1) = 1$
   b. $\frac{dN}{dt} = t^9 + t^{-11}$, $N(1) = \frac{3}{10}$
   c. $\frac{dL}{dt} = \sqrt{2t+9}$, $L(0) = 11$

2. Evaluate the following indefinite integrals.
   a. $\int \frac{1}{x+4} dx$
   b. $\int \frac{x}{x+4} dx$
   c. $\int \frac{x^2+1+x}{(x^2+1)x} dx$
   d. $\int \left(2x^{3/4} + 3x - 5\right) dx$
   e. $\int \frac{1}{\sqrt{9-x^2}} dx$
   f. $\int \frac{1}{9+x^2} dx$
   g. $\int e^{2t}(1-e^{-3t}) dt$
   h. $\int (s^2 - \sqrt{s}) ds$
   i. $\int \sec(y)(\sec(y) + \tan(y)) dy$

3. Evaluate the following definite integrals.
   a. $\int_0^{\pi} (\sin x + \cos x) dx$
   b. $\int_1^2 (s^2 - \sqrt{s}) ds$
   c. $\int_0^{\pi/8} \sec^2(2y) dy$
   d. $\int_{\ln 2}^{\ln 3} e^{2t}(1-e^{-3t}) dt$
   e. $\int_0^3 \frac{1}{x^2+9} dx$

4. Evaluate the following definite integral using only geometric considerations, i.e. interpret the integral as an area, draw a picture, and apply a well known geometric formula.

$$\int_0^4 \sqrt{4-(x-2)^2}\, dx$$

5. Approximate the definite integral $\int_0^2 (1-x^2)\, dx$ by calculating the Riemann Sum obtained by partitioning the interval $[0,2]$ into 8 *equal subintervals*, and choosing c_i to be the *left hand endpoint* of the i^{th} subinterval for $1 \le i \le 8$. **Do not evaluate this integral by using the Fundamental Theorem.**

6. Express the limit below as a definite integral, where $P = [x_0, x_1, \ldots, x_n]$ denotes a partition of the interval $[5,8]$ into n subintervals, $\Delta x_i = x_i - x_{i-1}$, and $c_i \in [x_{i-1}, x_i]$ for $1 \le i \le n$. **Do not evaluate this integral.**

$$\lim_{\|P\|\to 0} \sum_{i=1}^{n} \left(3c_i^5 + e^{c_i} + \sqrt{1+c_i^2}\right)\Delta x_i$$

7. Express the definite integral $\int_1^2 \frac{1}{\sqrt{x+\sqrt{x}}}\, dx$ as a limit of Riemann Sums. **Do not evaluate this integral.**

8. Let $g(x) = \int_1^{3x^2+x} (1+te^t)\, dt$. Determine $g'(x)$.

9. Determine the average value of the following functions on the indicated intervals.
   a. $\frac{1}{1+x^2}$ on $[0, \sqrt{3}]$
   b. $\sqrt{4-(x-2)^2}$ (Hint: first do problem 4.)

10. Suppose a particle travels along the graph $y = \ln(x)$ from $x = 1$ to $x = 2$. Set up a definite integral giving the length L of this path. Do not evaluate this integral.

11. (Do problem 10 before you try this one.) Suppose a particle travels along the graph $y = \ln(x)$ from $x = 1$ to $x = t$, where $t > 1$. Set up a definite integral giving the length $L(t)$ of this path. Determine the rate of change of this length, i.e. find $L'(t)$.

12. Determine the areas of the following plane regions.
   a. The region in the 1st quadrant bounded by $y = x^2$, $y = 1/x$, $y = 4$.
   b. The region in the 2nd quadrant bounded by $y = e^x$, and $x = -2$.
   c. The region in the 4th quadrant bounded by $y = \ln x$, $y = -2$. (Hint: Relate this to part (b).)

Some Indefinite Integrals

$\int x^n dx = \frac{x^{n+1}}{n+1} + C \quad (\text{for } n \neq -1)$

$\int x^{-1} dx = \ln|x| + C$

$\int e^x dx = e^x + C$

$\int a^x dx = \frac{a^x}{\ln a} + C$

$\int \cos x \, dx = \sin x + C$

$\int \sin x \, dx = -\cos x + C$

$\int \sec^2 x \, dx = \tan x + C$

$\int \csc^2 x \, dx = -\cot x + C$

$\int \sec x \tan x \, dx = \sec x + C$

$\int \csc x \cot x \, dx = -\csc x + C$

$\int \tan x \, dx = \ln|\sec x| + C$

$\int \cot x \, dx = -\ln|\csc x| + C$

$\int \frac{1}{1+x^2} dx = \tan^{-1} x + C$

$\int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1} x + C$

Standard Angles