Math 11B
Final Review Problems

Note: these problems represent *some* of the topics that were covered since Midterm 2. Bear in mind that the final will be comprehensive, with a slight emphasis on the most recent topics. See earlier review problems and solutions to Midterms 1 and 2 for review of earlier topics.

1. Use Integration by parts to prove the following formula: $\int x^n e^{ax}\,dx = \frac{1}{a}x^n e^{ax} - \frac{n}{a}\int x^{n-1}e^{ax}\,dx$.

2. Find the linear approximation to $f(x)=e^x$ for x near 2.

3. Find the n^{th} degree taylor polynomial of the following functions $f(x)$ for x near a.
   a. $f(x)=e^x$, $n=5$, $a=0$
   b. $f(x)=e^x$, $n=5$, $a=1$
   c. $f(x)=e^{-x^2}$, $n=10$, $a=0$ (Hint: substitue into your answer to part b)
   d. $f(x)=\sin x$, $n=7$, $a=0$
   e. $f(x)=\sqrt{1+x}$, $n=4$, $a=0$
   f. $f(x)=\ln x$, $n=4$, $a=1$

4. Use your answer to problem 3c above to estimate the value of the definite integral $\int_0^1 e^{-x^2}\,dx$. (Note you cannot calculate this definite integral directly by the FTC, so don't try.)

5. Use the remainder term $R_{n+1}(x)=\frac{f^{(n+1)}(c)}{(n+1)!}x^{n+1}$ for the n^{th} degree taylor polynomial to determine the smallest value of n which would guarantee that in estimating $f(x)=e^x$ for $x\in(-2,2)$, the error would not exceed 10^{-4}.

6. Solve the following initial value problems.
   a. $\frac{dy}{dx}=-2xy$, $y(1)=2$
   b. $\frac{dP}{dt}=Pe^{-t}$, $P(0)=1$
   c. $\frac{dP}{dt}=e^{-t}$, $P(0)=1$
   d. $\frac{dN}{dt}=(1+N^2)\cos(t)$, $N(0)=1$
   e. $\frac{dN}{dt}=1+N^2$, $N(0)=\sqrt{3}$
   f. $\frac{dy}{dx}=y(y-1)$, $y(0)=y_0$

7. Consider the autonomous differential equation $\frac{dy}{dx} = y^4 - y^2$

   a. Determine the equilibrium solutions
   b. Compute the eigenvalues of the equation
   c. Classify each of the equilibria as either stable, unstable, or semi-stable