(8.1.3) GENERAL SEPARABLE EQUATIONS.

RECALL THE MOST GENERAL SEPARABLE EQUATION IS

(1) $$\frac{dy}{dx} = f(x) \cdot g(y)$$

FROM WHICH WE GET

$$\int \frac{1}{g(y)}\, dy = \int f(x)\, dx$$

IF WE CAN FIND ANTIDERIVATIVES H AND F FOR $1/g$ AND f RESPECTIVELY ($\text{SO } H' = 1/g$ AND $F' = f$) THEN

$$H(y) = F(x) + C$$

IF FURTHER, WE CAN SOLVE THIS FOR y WE HAVE THE SOLUTION

(2) $$y(x) = H^{-1}(F(x) + C).$$

THE CONSTANT C IS DETERMINED BY INITIAL CONDITIONS.

LET US CHECK THAT THE PURPORTED SOLUTION (2) ACTUALLY SOLVES (1).

TAKING H OF BOTH SIDES OF (2) YIELDS

$$H(y(x)) = F(x) + C$$

DIFFERENTIATING WITH RESPECT TO x ON BOTH SIDES GIVES

$$H'(y(x)) \cdot y'(x) = F'(x)$$

USING $H' = 1/g$ AND $F' = f$ WE HAVE

$$\frac{1}{g(y(x))} \cdot y'(x) = f(x)$$

$$\therefore \quad y'(x) = g(y(x)) \cdot f(x)$$

PROVING THAT (2) DOES IN FACT SOLVE (1).

THIS JUSTIFIES THE PRACTICE OF 'FORMALLY' SEPARATING THE DIFFERENTIALS dy & dx.

Ex. $\frac{dy}{dx} = \frac{x+1}{y}$ $y(0) = 2$

Soln: $y(x) = \sqrt{(x+1)^2 + 3}$

Ex. $\frac{dy}{dx} = (y+1)e^{-x}$ $y(0) = 2$

Soln: $y(x) = 3\exp(1 - e^{-x}) - 1$

Ex. $\frac{dy}{dx} = \frac{y+1}{x-1}$ $y(2) = 5$

Soln: $y(x) = 6x - 7$

ALLOMETRIC GROWTH

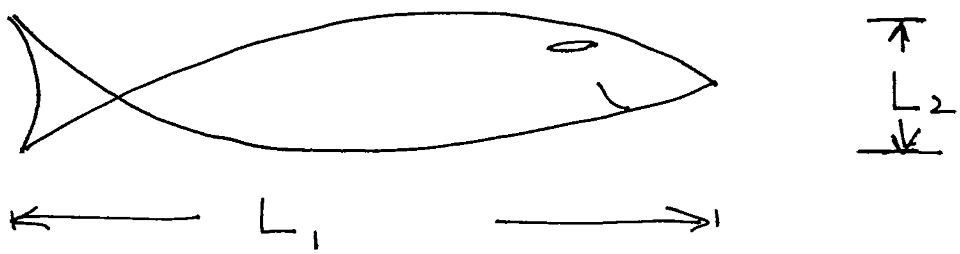
BIOLOGISTS OFTEN STUDY THE RELATIONSHIPS BETWEEN DIFFERENT PARTS OF AN ORGANISM SUCH AS ORGAN SIZES OR PHYSICAL DIMENSIONS OF AN INDIVIDUAL.

FOR INSTANCE LET $L_1(t)$ AND $L_2(t)$ DENOTE THE SIZES OF TWO ORGANS OF AN INDIVIDUAL AT TIME t.

WE SAY L_1 AND L_2 OBEY AN <u>ALLOMETRIC GROWTH</u> LAW IF THEIR SPECIFIC GROWTH RATES ARE PROPORTIONAL, i.e.

$$(*) \quad \frac{1}{L_1}\frac{dL_1}{dt} = K \cdot \frac{1}{L_2}\frac{dL_2}{dt}$$

FOR INSTANCE:



(NOTE ON 'SPECIFIC' RATES OF CHANGE $\frac{1}{y}\frac{dy}{dx}$.)

OBSERVE THAT THERE ARE THREE VARIABLES IN EQUATION (*): TWO DEPENDENT L_1, L_2 AND ONE INDEPENDENT: THIS IS NOT ENOUGH INFORMATION TO UNIQUELY DETERMINE $L_1(t)$ AND $L_2(t)$, HOWEVER, UPON ELIMINATING t FROM (*) WE HAVE

$$\frac{dL_1}{L_1} = k \frac{dL_2}{L_2}$$

$$\therefore \quad \ln|L_1| = k\ln|L_2| + C_1$$

$$\therefore \quad L_1 = C L_2^{k}$$

SEE EXAMPLE 8 ON P. 489

HW 10 8.1.4 (P.491)

2–16 even, 20 ab, 22 abcd,
26–30 even, 34, 36, 44, 46, 48