6.2.2 ANTIDERIVATIVES AND INDEFINITE INTEGRALS

RECALL THAT THE GENERAL ANTIDERIVATIVE OF $f(x)$ (i.e. THE ENTIRE FAMILY OF ANTIDERIVATIVES) IS OBTAINED BY CHOOSING A PARTICULAR ANTIDERIVATIVE $F(x)$ AND ADDING AN ARBITRARY CONSTANT C.

$$F(x) + C$$

IN PARTICULAR, WE MAY PICK $F(x) = \int_a^x f(u)\,du$, THUS

$$\int_a^x f(u)\,du + C$$

REPRESENTS ALL ANTIDERIVATIVES OF $f(x)$, UPON CHOOSING ALL POSSIBLE CONSTANTS C.

NOTATION: WE WRITE $\int f(x)\,dx$ TO STAND FOR THIS ENTIRE FAMILY OF ANTIDERIVATIVES. THUS

$$\int f(x)\,dx = F(x) + C$$

WHERE $F(x)$ IS <u>ANY</u> PARTICULAR ANTIDERIVATIVE OF $f(x)$.

THE EXPRESSION $\int f(x)\,dx$ IS CALLED THE <u>INDEFINITE INTEGRAL</u> OF $f(x)$ TO DISTINGUISH IT FROM THE DEFINITE INTEGRAL.

TABLE 6.1 (P. 366) GIVES SOME COMMON INDEFINITE INTEGRALS.

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C \qquad (n \neq -1)$$

$$\int \frac{1}{x}\,dx = \ln|x| + C$$

$$\int e^x\,dx = e^x + C$$

$$\int b^x\,dx = \frac{b^x}{\ln(b)} + C$$

$$\int \cos x \, dx = \sin x + C$$

$$\int \sin x \, dx = -\cos x + C$$

$$\int \sec^2 x \, dx = \tan x + C$$

$$\int \csc^2 x \, dx = -\cot x + C$$

$$\int \sec x \tan x \, dx = \sec x + C$$

$$\int \csc x \cot x \, dx = -\csc x + C$$

$$\int \tan x \, dx = \ln|\sec x| + C$$

$$\int \cot x \, dx = -\ln|\csc x| + C$$

$$\int \frac{1}{1+x^2} \, dx = \tan^{-1} x + C$$

$$\int \frac{1}{\sqrt{1-x^2}} \, dx = \sin^{-1} x + C$$

EACH OF THESE FORMULAS CORRESPONDS WITH A DIFFERENTIATION FORMULA.

FOR INSTANCE

$$\frac{d}{dx}\ln|x| = \begin{cases} \dfrac{d}{dx}\ln(x) & \text{if } x>0 \\ \dfrac{d}{dx}\ln(-x) & \text{if } x<0 \end{cases}$$

$$= \begin{cases} \dfrac{1}{x} & \text{if } x>0 \\ \dfrac{1}{-x}\cdot(-1) & \text{if } x<0 \end{cases}$$

$$= \frac{1}{x}$$

PROVING ~~THAT~~ $\int \frac{1}{x}\,dx = \ln|x| + C$

ALSO

$$\frac{d}{dx}\ln|\sec x| = \frac{1}{\cancel{\sec x}}\cdot \cancel{\sec x}\tan x$$

$$= \tan x$$

SHOWING $\int \tan x\,dx = \ln|\sec x| + C$.

<u>EX.</u> $\int \frac{1}{\cos^2 x - 1}\,dx = \int \frac{1}{\sin^2 x}\,dx = \int \csc^2 x\,dx$

$$= -\cot x + C$$

Ex. $\int \frac{x^2}{x^2+1}\,dx = \int \frac{(x^2+1)-1}{x^2+1}\,dx$

$$= \int \left(1 - \frac{1}{x^2+1}\right) dx$$

$$= x - \tan^{-1} x + C.$$