Math 11B
Midterm 2 Review Problems

Note: these problems are not a representation of all topics that may appear on midterm 2.

1. Evaluate the following indefinite integrals.
   a. $\int \frac{2}{3x+4}\,dx$
   b. $\int \frac{2x}{3x+4}\,dx$
   c. $\int xe^{x^2}\,dx$
   d. $\int x^3 e^{x^2}\,dx$
   e. $\int \frac{1}{x^2-9}\,dx$
   f. $\int \frac{1}{x^2+9}\,dx$
   g. $\int \frac{10x+6}{(x+1)^2}\,dx$
   h. $\int \frac{x^4+3x^3-2x^2+x+1}{x^2+2x+1}\,dx$
   i. $\int e^x \sin x\,dx$

2. Evaluate the following definite integrals.
   a. $\int_1^2 x^2 \ln(x^2)\,dx$
   b. $\int_e^{e^2} \frac{1}{x(\ln x)^2}\,dx$
   c. $\int_0^3 x^2 (x^3+100)^{1/3}\,dx$
   d. $\int_0^{\pi} e^x \sin x\,dx$
   e. $\int_0^{\pi/4} \tan x \sec^2 x\ e^{\tan x}\,dx$
   f. $\int_0^{\infty} \frac{1}{x^2+9}\,dx$
   g. $\int_0^1 \frac{1}{x^{2/3}}\,dx$
   h. $\int_1^3 (x-2)^{-2/3}\,dx$

3. Determine the partial fraction decomposition of the following rational functions. (Do not integrate.)
   a. $\dfrac{x^2+5}{(x-1)(x+2)^2}$
   b. $\dfrac{x^2+x+1}{(x+1)(x^2+2)}$

4. Determine the *form* of a partial fraction decomposition of the following rational functions. (Do not determine the constants.)
   a. $\dfrac{x^5+x^4+x^2-x+1}{(x-1)(2x+5)^2(3x-7)^3}$
   b. $\dfrac{x^{10}+x^7+x^3+x+3}{(x^2+x+1)(2x^2+5)^2(x^2-1)^3}$

5. For which p does $\displaystyle\int_0^1 \frac{1}{x^p}\,dx$ converge?

6. For which p does $\displaystyle\int_1^\infty \frac{1}{x^p}\,dx$ converge?

7. Determine the convergence or divergence of the following improper integrals. (Do not evaluate the integrals.)
   a. $\displaystyle\int_1^\infty \frac{1}{\sqrt{x+\sqrt{x}}}\,dx$ (Hint: Show that $\dfrac{1}{\sqrt{2x}} \le \dfrac{1}{\sqrt{x+\sqrt{x}}}$ for all $x \ge 1$.)
   b. $\displaystyle\int_0^1 \frac{1}{\sqrt{x+\sqrt{x}}}\,dx$ (Hint: Show that $\dfrac{1}{\sqrt{x+\sqrt{x}}} < \dfrac{1}{\sqrt{x}}$ for all $0 < x \le 1$.)

8. Solve the following initial value problem: $\begin{cases} \dfrac{dN}{dt} = te^t & t > 0 \\ N(0) = \dfrac{5}{2} \end{cases}$

9. Determine the average value of the following functions on the indicated intervals.
   a. $\ln x$ on $[1, e]$.
   b. $\dfrac{1}{1+x^2}$ on $[0, 1]$

10. Determine the area of the following plane regions.
    a. The region in the 1st quadrant bounded by $y = x^{-1/3}$, $y = 0$, $x = 0$, and $x = 1$.
    b. The region in the 2nd quadrant bounded by $y = e^x$, $y = 0$, and $x = 0$.
    c. The region in the 4th quadrant bounded by $y = \ln x$, $y = 0$, and $x = 0$.

Some Indefinite Integrals

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C \quad (\text{for } n \neq -1)$$

$$\int x^{-1} dx = \ln|x| + C$$

$$\int e^x dx = e^x + C$$

$$\int a^x dx = \frac{a^x}{\ln a} + C$$

$$\int \cos x \, dx = \sin x + C$$

$$\int \sin x \, dx = -\cos x + C$$

$$\int \sec^2 x \, dx = \tan x + C$$

$$\int \csc^2 x \, dx = -\cot x + C$$

$$\int \sec x \tan x \, dx = \sec x + C$$

$$\int \csc x \cot x \, dx = -\csc x + C$$

$$\int \tan x \, dx = \ln|\sec x| + C$$

$$\int \cot x \, dx = -\ln|\csc x| + C$$

$$\int \frac{1}{1+x^2} dx = \tan^{-1} x + C$$

$$\int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1} x + C$$

Standard Angles