Math 11B
HW #9
Solutions

(7.7.4) 2-16 even
20, 22, 26, 28, 30

GRADE: 4, 6, 8, 10, 12, 14, 16, 20, 22, 30

2.
$$f(0) + f'(0)x = \sin(0) + \cos(0)x = \underline{x}$$

$\boxed{4.}$
$$f(0) + f'(0)x = (0)^2 + 2(0) = \underline{0}$$

$\boxed{6.}$
$$f(0) + f'(0)x + \frac{f''(0)}{2}x^2 + \frac{f'''(0)}{6}x^3 + \frac{f''''(0)}{24}x^4$$
$$\frac{1}{1+0} + \frac{-1}{(1+0)^2}x + \frac{2}{2(1+0)^3}x^2 + \frac{-6}{6(1+0)^4}x^3 + \frac{24}{24(1+0)^5}x^4 = \underline{1 - x + x^2 - x^3 + x^4}$$

$\boxed{8.}$
$$e^{3(0)} + 3e^{3(0)}x + \frac{9}{2}e^{3(0)}x^2 + \frac{27}{6}e^{3(0)}x^3 = \underline{1 + 3x + \frac{9}{2}x^2 + \frac{9}{2}x^3}$$

$\boxed{10}$
$$\sqrt{1+0} + \frac{1}{2}\frac{1}{\sqrt{1+0}}x - \frac{1}{4}\frac{1}{(1+0)^{3/2}}\frac{1}{2}x^2 + \frac{3}{8}\frac{1}{(1+0)^{5/2}}\frac{1}{6}x^3 = \underline{1 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{1}{16}x^3}$$

$\boxed{12}$
$$1 + \frac{1}{(1-0)^2}x + \frac{2}{2(1-0)^3}x^2 + \frac{6}{6(1-0)^4}x^3 = 1 + x + x^2 + x^3 = 1.111$$
$$f(.1) = \frac{1}{.9} = 10/9 \qquad f(.1) - p(.1) = \underline{.0001}$$

$\boxed{14.}$
$$1 - x + \frac{x^2}{2} - \frac{x^3}{6} + \frac{x^4}{24} - \frac{x^5}{120} = .74081725$$
$$e^{-.3} = .74081822$$
$$f(.3) - p(.3) = \underline{9.7 \times 10^{-7}}$$

$\boxed{16.}$
$$x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} = .18226$$
$$\ln(1+.2) = .18232$$
$$|f(.2) - p(.2)| = \underline{5.49 \times 10^{-5}}$$

$\boxed{20}$

$$\ln(1) + \frac{1}{(1)}(x-1) - \frac{1}{(1)^2}\frac{1}{2}(x-1)^2 + \frac{2}{6}(x-1)^3 = (x-1) - \frac{1}{2}(x-1)^2 + \frac{1}{3}(x-1)^3$$

$$p(2) = 5/6 \approx .8333$$

$$\ln(2) \approx .6931$$

$$|f(2) - p(2)| = .1402$$

$\boxed{22}$

$$(-1)^{1/3} + \frac{1}{3}(-1)^{-2/3}(x+1) + \frac{-2}{9 \cdot 2}(-1)^{-5/3}(x+1)^2 + \frac{10}{27 \cdot 6}(-1)^{-8/3}(x+1)^3$$

$$= -1 + \frac{1}{3}(x+1) + \frac{1}{9}(x+1)^2 + \frac{5}{81}(x+1)^3$$

$$p(-.9) = -.96549$$

$$f(-.9) = -.965489$$

$$|f(-.9) - p(-.9)| = 4.44 \times 10^{-6}$$

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a) Expand around 0 $\quad f(0) = 0 \quad f'(0) = \frac{-a(0)}{(k+(0))^2} + \frac{a}{k+(0)} = \frac{a}{k}$

So $\quad p(R) = 0 + \frac{a}{k}R = \frac{a}{k}R$

b) Expand around k $\quad f(k) = \frac{ak}{k+k} = \frac{a}{2} \quad f'(k) = \frac{-a(k)}{(k+(k))^2} + \frac{a}{k+k}$

$$= \frac{-a}{4k} + \frac{a}{2k} = \frac{a}{4k}$$

$\cancel{f(R) \approx}$ $p(R) = \frac{a}{2} + \frac{a}{4k}(R-k)$

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$f^{(n+1)} = \begin{cases} \sin(x) \\ \cos(x) \end{cases}$, on $[0, 1]$ these have max values of $1, \sin(1) \approx .84$

$$R_{n+1}(x) = \frac{[1, .8]}{(n+1)!}(1)^{n+1} = \frac{[1, .8]}{(n+1)!} < 10^{-2} \Rightarrow n = 4$$

$\boxed{30}$ $f^{(n+1)}(x) = \frac{1}{n!(x+1)^{n+1}}$ max at $x=0$ with value $\frac{1}{n!}$

Therefore $\quad R_{n+1}(x) = \frac{1}{n!(n+1)!}(x)^{n+1} = \frac{1}{n!(n+1)!}(10^{-1})^{n+1}$ check $n=1$ $R_2 = 5 \times 10^{-2} > 10^{-3}$

$n=2$ $R_3 = \frac{1}{12} \times 10^{-3} < 10^{-3}$

$\underline{n=2}$