Math 11B - HW 8

7.6.1 (P.457) 2-22 even

GRADED: 2, 4, 6, 8, 10, 12, 16, 18, 20, 22

$\boxed{2}$ p453 prop. 13 let $a=4$ gives:

$$\int \frac{dx}{16+x^2} = \frac{1}{4}\arctan\frac{x}{4} + C$$

$\boxed{4}$ Property 25, let $a=2$ gives:

$$\int \sin 2x \cos 2x \, dx = \frac{1}{4}\sin^2(2x) + C$$

$\boxed{6}$

$$\int_0^{\pi/4} e^{-x}\cos(2x)\,dx = \left[\frac{e^{-x}}{5}(2\sin 2x - \cos 2x)\right]_0^{\frac{\pi}{4}}$$

$$= \frac{2e^{-\pi/4}}{5} + \frac{1}{5}$$

(By property 31, $a=-1$, $b=2$)

$\boxed{8}$

$$\int_e^{e^2} \frac{dx}{x\ln x} = \left[\ln(\ln x)\right]_e^{e^2}$$

$$= \ln 2$$

(By property 36)

$\boxed{10}$

$$\int_1^2 x\ln(2x-1)\,dx = \frac{1}{2}\int_1^3\left(\frac{u+1}{2}\right)\ln u\,du$$

$u = 2x-1$

$\Rightarrow dx = \frac{1}{2}du$

$2 \mapsto 3 \quad 1 \mapsto 1$

$$= \frac{1}{4}\left\{\int_1^3 u\ln u\,du + \int_1^3 \ln u\,du\right\}$$

$$= \frac{1}{4}\left\{\left[u^2\left(\frac{\ln u}{2} - \frac{1}{4}\right)\right]_1^3 + \left[u\ln u - u\right]_1^3\right\}$$

$$= \frac{1}{4}\left\{\frac{15}{2}\ln 3 - \frac{9}{4} + \frac{1}{4} + (-3-(-1))\right\}$$

$$= \frac{1}{4}\left(\frac{15}{2}\ln 3 - \frac{16}{4}\right)$$

$$= \frac{15}{8}\ln 3 - 1$$

(By PROPERTIES 32 and 34)

$\boxed{12}$ $\int (x+1)^2 e^{-2x}\,dx = \int u^2 e^{-2u} e^2\,du$

$u = x+1$
$du = dx$

$$= e^2\left\{-\tfrac{1}{2}u^2 e^{-2u} + \int u e^{-2u}\,du\right\}$$

$$= e^2\left[-\tfrac{1}{2}u^2 e^{-2u} + \frac{e^{-2u}}{4}(-2u-1)\right]$$

$$= -\tfrac{1}{2}(x+1)^2 e^{-2x} + \tfrac{1}{4}e^{-2x}(-2(x+1)-1)$$

$$= -\frac{e^{-2x}}{4}(2x^2+6x+5) + C$$

By propeties 28 + 29.

14 $\int \frac{x^2}{4x^2+4x+1}\,dx = \int \frac{x^2}{(2x+1)^2}\,dx$

$u = x$ $\qquad v' = \cancel{x}\ \frac{x}{(2x+1)^2}$

$u' = 1$ $\qquad v = \tfrac{1}{4}\left(\frac{1}{(2x+1)} + \ln(2x+1)\right)$ $\qquad$ By prop. 11

First, calculate $\tfrac{1}{4}\int \frac{1}{2x+1} + \ln(2x+1)\,dx$

$$= \tfrac{1}{4}\left(\tfrac{1}{2}\ln(2x+1) + \tfrac{1}{2}(2x+1)\ln(2x+1) - \tfrac{1}{2}(2x+1)\right)$$

$$= \tfrac{1}{8}\ln(2x+1) + \tfrac{1}{8}(2x+1)\ln(2x+1) - \tfrac{1}{8}(2x+1)$$

So $\int \frac{x^2}{4x^2+4x+1}\,dx = uv - \int u'v$

$$= \frac{x}{4}\left(\frac{1}{(2x+1)} + \ln(2x+1)\right) - \tfrac{1}{4}\int \frac{1}{2x+1} + \ln(2x+1)\,dx$$

$$= \tfrac{1}{8}\left(\frac{2x}{2x+1} + 3\ln(2x+1) + (2x+1)\ln(2x+1) - (2x+1)\right) + C$$

$\boxed{16}$ $\int \frac{1}{\sqrt{16-9x^2}}\,dx = \int \frac{1}{\sqrt{9(\frac{16}{9}-x^2)}}\,dx$

$= \frac{1}{3}\int \frac{1}{\sqrt{\frac{16}{9}-x^2}}\,dx$

$= \frac{1}{3}\arcsin\left(\frac{3x}{4}\right) + C$

By property 15, $a = \frac{4}{3}$

$\boxed{18}$ $\int (x-1)^2 e^{2x}\,dx = \int u^2 e^{2u+2}\,du$

$u = x-1$ $\quad = e^2 \int u^2 e^{2u}\,du$

$= e^2\left(\frac{1}{2}u^2 e^{2u} - \int u e^{2u}\,du\right) + C$

(By 29 with $n=2$, $a=2$)

$= e^2\left(\frac{1}{2}u^2 e^{2u} - \frac{e^{2u}}{4}(2u-1)\right) + C$

(By 28) $\quad = \frac{e^{2u+2}}{4}\left(2u^2 - 2u + 1\right) + C$

$= \frac{e^{2x}}{4}\left(2(x-1)^2 - 2(x-1) + 1\right) + C$

$= \frac{e^{2x}}{4}\left(5 - 6x + 2x^2\right) + C$

$\boxed{20}$ $\int_1^e (x+2)^2 \ln x\,dx = \int_1^e (x^2+4x+4)\ln x\,dx$

$= \int_1^e x^2 \ln x\,dx + 4\int_1^e x\ln x\,dx + 4\int_1^e \ln x\,dx$

$= Q7 + 4\left[x^2\left(\frac{\ln x}{2} - \frac{1}{4}\right)\right]_1^e + 4\left[x\ln x - x\right]_1^e$

$$= \left(\frac{1}{9} + \frac{2}{9}e^{3}\right) + \left(1+e^{2}\right) + 4$$

$$= \frac{2}{9}e^{3} + e^{2} + \frac{46}{9}$$

$\boxed{22}$ $$\int \frac{3}{x^2-4x+8}\,dx = 3\int \frac{1}{(x+2)^2+4}\,dx$$

$u = x-2$
$du = dx$

$$= 3\int \frac{1}{u^2+4}\,du$$

$$= 3\left(\frac{1}{2}\tan^{-1}\left(\frac{u}{2}\right)\right) + C$$

By property 13

$$= \frac{3}{2}\tan^{-1}\left(\frac{x-2}{2}\right) + C$$