MATH 11B – HW 6 SOLUTIONS

Section 7.2.1

2. $\int 3x\cos x\,dx = 3\left[x\sin x - \int \sin x\,dx\right] = 3x\sin x + 3\cos x + C$

4. $\int 3x\cos(1-x)\,dx = -3x\sin(1-x) + \int 3\sin(1-x)\,dx = -3x\sin(1-x) + 3\cos(1-x) + C$

6. $\int x\sin 2x\,dx = -\frac{1}{2}x\cos 2x + \int \frac{1}{2}\cos 2x\,dx = -\frac{1}{2}x\cos 2x + \frac{1}{4}\sin 2x + C$

8. $\boxed{\int 2xe^{-x}\,dx = -2xe^{-x} + \int 2e^{-x}\,dx = -2xe^{-x} - 2e^{-x} + C}$

10. $\int 2x^2e^{-x}\,dx = -2x^2e^{-x} + \int 4xe^{-x}\,dx = -2x^2e^{-x} - 4xe^{-x} + \int 4e^{-x}$

$= -2x^2e^{-x} - 4xe^{-x} - 4e^{-x} + C$

12. $\boxed{\int x^2\ln x\,dx = \frac{1}{3}x^3\ln x - \int \frac{1}{3}x^3\frac{1}{x}\,dx = \frac{1}{3}x^3\ln x - \frac{1}{9}x^3 + C}$

14. $\boxed{\int x^2\ln x^2 = \frac{1}{3}x^3\ln x^2 - \int \frac{1}{3}x^3\frac{2x}{x^2}\,dx = \frac{1}{3}x^3\ln x^2 - \frac{2}{9}x^3 + C}$

16. $\int x\csc^2 x\,dx = -x\cot x + \int \cot x\,dx = -x\cot x - \ln|\csc x| + C$

18. $\int_0^{\pi/4} x\cos x\,dx = x\sin x\Big|_0^{\pi/4} - \int_0^{\pi/4}\sin x\,dx = x\sin x\Big|_0^{\pi/4} + \cos x\Big|_0^{\pi/4}$

$= \frac{\pi}{4}\left(\frac{\sqrt{2}}{2}\right) - 0 + \frac{\sqrt{2}}{2} - 1 = \left(\frac{4+\pi}{4}\right)\frac{\sqrt{2}}{2} - 1$

20. $\boxed{\int_1^e \ln x^2\,dx = x\ln x^2\Big|_1^e - \int_1^e \frac{2x}{x^2}x\,dx = x\ln x^2 - 2x\Big|_1^e = 2e - 0 - 2e + 2 = 2}$

22. $\int_1^4 \sqrt{x}\ln\sqrt{x}\,dx = \frac{1}{2}\int_1^4 \sqrt{x}\ln x\,dx = \frac{1}{3}x^{\frac{3}{2}}\ln x\Big|_1^4 - \frac{1}{3}\int_1^4 x^{\frac{1}{2}}\,dx$

$= \frac{1}{3}x^{\frac{3}{2}}\ln x - \frac{2}{9}x^{\frac{3}{2}}\Big|_1^4$

$= \frac{1}{3}(4)^{3/2}\ln 4 - 0 - \frac{2}{9}(4)^{\frac{3}{2}} + \frac{2}{9}$

$= \frac{8}{3}\ln 4 - \frac{14}{9}$

24. $\int_0^3 x^2e^{-x}\,dx = -x^2e^{-x} - 2xe^{-x} - 2e^{-x}\Big|_0^3 = -3^2e^{-3} + 0 - 2(3)e^{-3} + 0 - 2e^{-3} + 2e^0$

$= 2 - 17e^{-3}$

26. $\int_0^{\pi/6} e^x \cos x\,dx = e^x \cos x\Big|_0^{\pi/6} + \int_0^{\pi/6} e^x \sin x\,dx = e^x \cos x\Big|_0^{\pi/6} + e^x \sin x\Big|_0^{\pi/6} - \int e^x \cos x\,dx$

$2\int_0^{\pi/6} e^x \cos x\,dx = e^x \cos x\Big|_0^{\pi/6} + e^x \sin x\Big|_0^{\pi/6}$

$\int_0^{\pi/6} e^x \cos dx = \frac{1}{2} e^{\pi/6} \frac{\sqrt{3}}{2} - \frac{1}{2} + \frac{1}{4} e^{\pi/6} = \frac{1+\sqrt{3}}{4} e^{\pi/6} - \frac{1}{2}$

28. $\int e^{-2x} \sin \frac{x}{2}\,dx = -2e^{-2x} \cos \frac{x}{2} - \int 4e^{-2x} \cos \frac{x}{2}\,dx$

$= -2e^{-2x} \cos \frac{x}{2} - 8e^{-2x} \sin \frac{x}{2} + 16\int e^{-2x} \sin \frac{x}{2}$

$-15\int e^{-2x} \sin \frac{x}{2}\,dx = -2e^{-2x} \cos \frac{x}{2} - 8e^{-2x} \sin \frac{x}{2} + C$

$\int e^{-2x} \sin \frac{x}{2}\,dx = \frac{2}{15} e^{-2x} \cos \frac{x}{2} + \frac{8}{15} e^{-2x} \sin \frac{x}{2} + C$

30 $\int \cos(\ln x)\,dx = x\cos(\ln x) + \int x \sin(\ln x) \frac{1}{x}\,dx$

$= x\cos(\ln x) + x\sin(\ln x) - \int x\cos(\ln x)\frac{1}{x}$

$2\int \cos(\ln x)\,dx = x\cos(\ln x) + x\sin(\ln x)$

$\int \cos(\ln x)\,dx = \frac{x}{2}\left(\cos(\ln x) + \sin(\ln x)\right)$

32 $\int \sin^2 x\,dx = \int \sin x \sin x\,dx = -\sin x\cos x + \int \cos^2 x\,dx$

$= -\sin x\cos x + \int 1 - \sin^2 x\,dx$

$= -\sin x\cos x + \int 1\,dx - \int \sin^2 x\,dx$

$2\int \sin^2 x\,dx = -\sin x\cos x + x + C$

$\int \sin^2 x\,dx = -\frac{1}{2}\sin x\cos x + \frac{1}{2}x + C$

34 $\int \arccos x\,dx = \int 1\cdot \arccos x\,dx = x\arccos x - \int x \frac{d}{dx}(\arccos x)\,dx$

$u = 1-x^2$

$= x\arccos x + \int \frac{x}{\sqrt{1-x^2}}\,dx$

$\int \frac{x}{\sqrt{1-x^2}}\,dx = \int \frac{-1}{2\sqrt{u}}\,du = \frac{-1}{4}\sqrt{u}$

$\int \arccos x\,dx = x\arccos x - \frac{1}{4}\sqrt{1-x^2}$

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$$\int \sin x \cos^3 x \, e^{1-\sin^2 x} \, dx = \int \sin x \cos^3 x \, e^{\cos^2 x} \qquad u = \cos^2 x \quad du = -2\sin x \cos x \, dx$$

$$= \int -\tfrac{1}{2} u e^{u} = -\tfrac{1}{2} u e^{u} + \int \tfrac{1}{2} e^{u} = -\tfrac{1}{2} u e^{u} + \tfrac{1}{2} e^{u}$$

$$= \tfrac{1}{2} \cos^2 x \, e^{\cos^2 x} + \tfrac{1}{2} e^{\cos^2 x} + C$$

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$$\int x^3 \ln(x^2+1) \qquad u = x^2+1 \qquad du = 2x\,dx$$

$$= \int \tfrac{1}{2}(u-1)\ln u \, du = \left(\tfrac{1}{4}u^2 - \tfrac{1}{2}u\right)\ln u - \int \frac{\tfrac{1}{4}u^2 - \tfrac{1}{2}u}{u}\, du$$

$$= \left(\tfrac{1}{4}u^2 - \tfrac{1}{2}u\right)\ln u - \tfrac{1}{8}u^2 - \tfrac{1}{2}u + C$$

$$= \left[\tfrac{1}{4}(x^2+1)^2 - \tfrac{1}{2}(x^2+1)\right]\ln(x^2+1)$$

$$- \tfrac{1}{8}(x^2+1)^2 - \tfrac{1}{2}(x^2+1) + C$$

Section 7.3.3

2. $\int x e^{-2x^2} dx$ $\quad u = -2x^2 \quad du = -4x\,dx$

$= \int -\frac{1}{4} e^u = -\frac{1}{4} e^u = -\frac{1}{4} e^{-2x^2} + C$

4 $\int \frac{1}{\csc x \sec x} dx = \int \sin x \cos x \, dx = \int \frac{d}{dx}\left(\frac{1}{2}\sin^2 x\right) = \frac{1}{2}\sin^2 x + C$

$= \int \frac{d}{dx}\left(-\frac{1}{2}\cos^2 x\right) = -\frac{1}{2}\cos^2 x + C$

6) $\int 2x^2 \sin x \, dx = -2x^2 \cos x + \int 4x \cos x \, dx = -2x^2\cos x + 4x \sin x - \int 4 \sin x \, dx$

$= -2x^2 \cos x + 4x \sin x + 4\cos x + C$

8 $\int \frac{1}{x^2+5} = \int \frac{1}{5}\left(\frac{1}{\frac{1}{5}x^2+1}\right) dx$ $\quad u = \frac{1}{\sqrt{5}} x \quad du = \frac{1}{\sqrt{5}} dx$

$= \frac{\sqrt{5}}{5} \int \frac{1}{u^2+1} du = \frac{\sqrt{5}}{5} \arctan(u) = \frac{\sqrt{5}}{5} \arctan\left(\frac{1}{\sqrt{5}} x\right) + C$

10 $\int \frac{1}{x^2+3} = \int \frac{1}{3}\left(\frac{1}{\frac{1}{3}x^2+1}\right) dx$ $\quad u = \frac{1}{\sqrt{3}} x \quad du = \frac{1}{\sqrt{3}} dx$

$= \frac{\sqrt{3}}{3} \int \frac{1}{u^2+1} du = \frac{\sqrt{3}}{3} \arctan(u) = \frac{\sqrt{3}}{3} \arctan\left(\frac{1}{\sqrt{3}} x\right) + C$

12 $\int \frac{x+2}{x^2+2} = \int \frac{x}{x^2+2} + \int \frac{2}{x^2+2}$

$\int \frac{x}{x^2+2} \rightarrow u = x^2+2$, $du = 2x\,dx$ $\quad \int \frac{1}{2u} du = \frac{1}{2}\ln u = \frac{1}{2}\ln(x^2+2)$

$\int \frac{2}{x^2+2} = \sqrt{2} \arctan\left(\frac{1}{\sqrt{2}} x\right)$

$\int \frac{x+2}{x^2+2} = \frac{1}{2}\ln(x^2+2) + \sqrt{2}\arctan\left(\frac{1}{\sqrt{2}} x\right)$

14 $\int x^3 e^{-x^2/2} dx$ $\quad u = -x^2/2 \quad du = -x\,dx$

$= \int 2u e^u du = 2ue^u - \int 2e^u du = 2ue^u - 2e^u$

$= -x^2 e^{-x^2/2} - 2e^{-x^2/2}$

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$$\int \frac{\sin^2 x - \cos^2 x}{(\sin x - \cos x)^2}dx = \int \frac{(\sin x + \cos x)(\sin x - \cos x)}{(\sin x - \cos x)(\sin x - \cos x)}dx = \int \frac{\sin x + \cos x}{\sin x - \cos x}dx$$

$u = \sin x - \cos x$

$du = (\cos x + \sin x)dx$

$$= \int \frac{1}{u}du = \ln u + C = \ln(\sin x - \cos x) + C$$

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$$\int_1^2 \ln(x^2 e^x)dx = \int_1^2 \ln x^2 dx + \int_1^2 \ln e^x dx = \int_1^2 2\ln x\, dx + \int_1^2 x\, dx$$

$$= 2x\ln x - 2x + \tfrac{1}{2}x^2 \Big|_1^2$$

$$= 2(2)\ln 2 - 2(2) + \tfrac{1}{2}(2)^2$$

$$- 0 + 2(1) - \tfrac{1}{2}$$

$$= 4\ln 2 - \tfrac{1}{2}$$