MATH 11B

HW 5 SOLUTIONS

GRADED: 2, 10, 26, 28, 34, 36, 42, 44, 46, 56

2/ $u = x^3 + 1 \Rightarrow \frac{du}{dx} = 3x^2 \Rightarrow du = 3x^2\,dx$

$\Rightarrow \int 3x^2 \sqrt{x^3+1}\,dx = \int u^{\frac{1}{2}}\,du = \frac{2}{3} u^{\frac{3}{2}} = \frac{2}{3}(x^3+1)^{\frac{3}{2}} + C$

4/ $u = 4 + x^4 \Rightarrow \frac{du}{dx} = 4x^3 \Rightarrow \frac{du}{4} = \cancel{4}x^3\,dx$

$\Rightarrow \int x^3 \sqrt{4+x^4}\,dx = \int \frac{1}{4} u^{\frac{1}{2}}\,du = \frac{1}{6} u^{\frac{3}{2}} = \frac{1}{6}(4+x^4)^{\frac{3}{2}} + C$

6/ $u = 1 - 2x \Rightarrow \frac{du}{dx} = -2 \Rightarrow \frac{du}{-2} = dx$

$\Rightarrow \int 5 \sin(1-2x)\,dx = \int -\frac{5}{2} \sin u\,du = -\frac{5}{2}(-\cos u)$

$= \frac{5}{2} \cos(1-2x) + C$

8/ $u = x^2 - 1 \Rightarrow \frac{du}{dx} = 2x \Rightarrow \frac{du}{2} = x\,dx$

$\Rightarrow \int x \cos(x^2-1)\,dx = \int \frac{1}{2} \cos(u)\,du = \frac{1}{2} \sin(x^2-1) + C$

10/ $u = 1 - x \Rightarrow \frac{du}{dx} = -1 \Rightarrow \frac{du}{-1} = dx$

$\Rightarrow \int 3e^{1-x}\,dx = \int -3e^u\,du = -3e^u = -3e^{1-x} + C$

12/ $u = 1 - 3x^2 \Rightarrow \frac{du}{dx} = -6x \Rightarrow \frac{du}{-6} = x\,dx$

$\Rightarrow \int x e^{1-3x^2}\,dx = \int -\frac{1}{6} e^u\,du = -\frac{1}{6} e^{1-3x^2} + C$

14/ $u = 3 - x^2 \Rightarrow \frac{du}{dx} = -2x \Rightarrow \frac{du}{-1} = 2x\,dx$

$\Rightarrow \int \frac{2x}{3-x^2}\,dx = \int -\frac{1}{u}\,du = -\ln u = -\ln|3-x^2| + C$

16. $u = 5 - x \Rightarrow \frac{du}{dx} = -1 \Rightarrow \frac{du}{-1} = dx$

$\int \frac{1}{5-x}\,dx = \int -\frac{1}{u}\,du = -\ln u = -\ln(5-x) + C$

18. $u = 4 - x \Rightarrow \frac{du}{dx} = -x \Rightarrow \frac{du}{-1} = dx$

$\Rightarrow \int \sqrt{4-x}\,dx = \int -u^{\frac{1}{2}}\,du = -\frac{2}{3}u^{\frac{3}{2}} \cancel{+C} = -\frac{2}{3}(4-x)^{\frac{3}{2}} + C$

20. $u = x^3 - 3x^2 + 3 \Rightarrow \frac{du}{dx} = 3x^2 - 6x \Rightarrow \frac{du}{3} = (x^2 - 2x)\,dx$

$\int (x^2 - 2x)\sqrt{x^3 - 3x^2 + 3}\,dx \Rightarrow \int \frac{1}{3}u^{\frac{1}{2}}\,du = \frac{2}{9}(x^3 - 3x^2 + 3)^{\frac{1}{2}} + C$

22. $u = x^3 - 3x + 1 \Rightarrow \frac{du}{dx} = 3x^2 - 3 \Rightarrow \frac{du}{3} = (x^2 - 1)\,dx$

$\Rightarrow \int \frac{x^2 - 1}{x^3 - 3x + 1}\,dx = \int \frac{1}{3}\frac{1}{u}\,du = \frac{1}{3}\ln u = \frac{1}{3}\ln|x^3 - 3x + 1| + C$

24. $u = x^4 - 4x \Rightarrow \frac{du}{dx} = 4x^3 - 4 \Rightarrow \frac{du}{4} = (x^3 - 1)\,du$

$\Rightarrow \int \frac{x^3 - 1}{x^4 - 4x}\,dx = \int \frac{1}{4}\frac{1}{u}\,du = \frac{1}{4}\ln|x^4 - 4x| + C$

26. $u = \sin x \Rightarrow \frac{du}{dx} = \cos x \Rightarrow du = \cos x\,dx$

$\Rightarrow \int \cos x\, e^{\sin x}\,dx = \int e^u\,du = e^u = e^{\sin x} + C$.

28. $u = \tan x \Rightarrow du = \sec^2 x\,dx$

$\Rightarrow \int \sec^2 x\, e^{\tan x}\,dx = \int e^u\,du = e^{\tan x} + C$

30. $u = 2x - 1 \Rightarrow \frac{du}{dx} = 2 \Rightarrow \frac{du}{2} = dx$

$\Rightarrow \int \cos(2x-1)\,dx = \int \frac{1}{2}\cos u\,du = \frac{1}{2}\sin(2x-1) + C$

32. $u = \sin x \Rightarrow \frac{du}{dx} = \cos x \Rightarrow du = \cos x \, dx$

$\Rightarrow \int \sin^3 x \cos x \, dx = \int u^3 \, du = \frac{1}{4} u^4 = \frac{1}{4} \sin^4 x + C$

34. $u = \ln|x-3| \Rightarrow \frac{du}{dx} = \frac{1}{x-3} \Rightarrow du = \frac{1}{x-3} \, dx$

$\Rightarrow \int \frac{dx}{(x-3)(\ln(x-3))} = \int \frac{1}{u} \, du = \ln(u) = \ln\big(\ln|x-3|\big) + C$

36. Set $u = 1 + \ln x$. Note $\Rightarrow \ln x = u - 1 \quad (*)$

$\frac{du}{dx} = \frac{1}{x} \Rightarrow du = \frac{1}{x} \, dx$

$\Rightarrow \int \sqrt{1+\ln x} \, \frac{\ln x}{x} \, dx = \int \sqrt{u} \, \ln x \, du \quad \big(\text{use } (*)\big)$?..

$= \int \sqrt{u}\,(u-1) \, du = \int u^{\frac{3}{2}} - u^{\frac{1}{2}} \, du$

$= \frac{2}{5} u^{\frac{5}{2}} - \frac{2}{3} u^{\frac{3}{2}} =$

$= \frac{2}{5} (1+\ln x)^{\frac{5}{2}} - \frac{2}{3} (1+\ln x)^{\frac{3}{2}} + C$

42. $u = g(x) \Rightarrow \frac{du}{dx} = g'(x) \Rightarrow du = g'(x) \, dx$

$\Rightarrow \int \frac{g'(x)}{(g(x))^2 + 1} \, dx = \int \frac{1}{u^2+1} \, du = \tan^{-1}(u) = \tan^{-1}(g(x)) + C$

44. $u = x^3 + 2 \Rightarrow \frac{du}{dx} = 3x^2 \Rightarrow \frac{du}{3} = x^2 \, dx$

$\Rightarrow \int_1^2 x^2 \sqrt{x^3+2} \, dx = \int_{1^3+2}^{2^3+2} \frac{1}{3} \sqrt{u} \, du = \left[\frac{2}{9} u^{\frac{3}{2}} \right]_3^{10}$

$= \frac{2}{9}\left(10^{\frac{3}{2}} - 3^{\frac{3}{2}}\right)$

46. $\int_0^2 \frac{2x}{\sqrt{4x^2+3}} \, dx$

$u = 4x^2 + 3 \Rightarrow \frac{du}{dx} = 8x \Rightarrow \frac{du}{4} = 2x \, dx$

$\int_0^2 \frac{2x}{\sqrt{4x^2+3}} \, dx = \int_3^{19} \frac{1}{4} \frac{1}{u} \, du = \frac{1}{4} \left[\ln u \right]_3^{19} = \frac{1}{4} \ln\left|\frac{19}{3}\right|$

48 $u = e^x - 3 \Rightarrow \frac{du}{dx} = e^x \quad du = e^x \, dx$

$\Rightarrow \int_{\ln 4}^{\ln 7} \frac{e^x}{(e^x-3)^2} \, dx = \int_1^4 \frac{1}{u^2} \, du = -\left[\frac{1}{u} \right]_1^4 = \frac{3}{4}$

50 $u = \sin x \Rightarrow \frac{du}{dx} = \cos x \quad du = \cos x \, dx$

$\Rightarrow \int_{-\pi/6}^{\pi/6} \sin^2 x \cos x \, dx = \int_{-\frac{1}{2}}^{\frac{1}{2}} u^2 \, du = \frac{1}{3} \left[u^3 \right]_{-\frac{1}{2}}^{\frac{1}{2}} = \frac{1}{12}$

52 $u = \cos x \Rightarrow \frac{du}{dx} = -\sin x \Rightarrow \frac{du}{-1} = \sin x \, dx$

$\Rightarrow \int_0^{\pi/3} \frac{\sin x}{\cos^2 x} \, dx = \int_1^{\frac{1}{2}} -\frac{1}{u^2} \, du = \left[\frac{1}{u} \right]_1^{\frac{1}{2}} = 1$

54 $u = x+2 \Rightarrow \frac{du}{dx} = 1 \Rightarrow du = dx$

$\Rightarrow \int_2^4 \frac{u-2}{u} \, du = \int_2^4 1 - \frac{2}{u} \, du = \left[u - 2\ln|u| \right]_2^4$

$= (4 - 2\ln 4) - (2 - 2\ln 2)$

$= 2 + 2(\ln 2 - \ln 4) = 2 - 2\ln(2)$

56 $u = \ln(x^2+1) \Rightarrow \frac{du}{dx} = \frac{2x}{x^2+1} \Rightarrow \frac{du}{2} = \left(\frac{x}{x^2+1} \right) dx$

$\Rightarrow \int_1^2 \frac{x}{(x^2+1)\ln(x^2+1)} \, dx = \int_{\ln 1}^{\ln 5} \frac{1}{u} \, du = \left[\ln u \right]_0^{\ln 5}$

$= \ln(\ln(5))$

58 $\int_0^2 x\sqrt{4-x^2} \, dx$

$u = 4 - x^2 \Rightarrow \frac{du}{dx} = -2x \Rightarrow \frac{du}{-2} = x \, dx$

$\Rightarrow \int_4^0 -\frac{1}{2}\sqrt{u} \, du = \left[-\frac{1}{3} u^{\frac{3}{2}} \right]_4^0 = \frac{8}{3}$