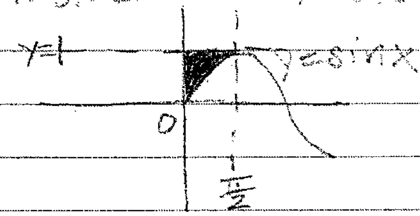
Math 11B
HW 4 Solutions

p.392 #2-16 even, 18, 22, 24, 26, 28, 32, 54, 56, 58, 60, 62

(2) $y=\sin x,\ y=1,\ x=0,\ x=\frac{\pi}{2}$



$A={}_0\int^{\pi/2}(1-\sin x)\,dx$

$=x+\cos x\,\Big|_0^{\pi/2}=\frac{\pi}{2}+0-(0+1)$

$A=\frac{\pi}{2}-1$

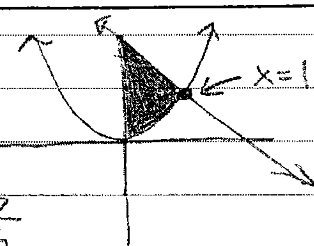
(4) $y=x^2,\ y=2-x^4$ $\quad x^2=2-x^4\Rightarrow x^4+x^2-2=0\Rightarrow(x^2+2)(x^2-1)=0$

x is real, so $x^2\neq-2$. $x^2=1\Rightarrow x=\pm1$

$A={}_{-1}\int^{1}2-x^4-x^2\,dx$ $\quad x^2\le1\le2-x^4$ on $[-1,1]$

$A=2x-\frac{1}{5}x^5-\frac{1}{3}x^3\,\Big|_{-1}^{1}=2-\frac{1}{5}-\frac{1}{3}-\left(-2+\frac{1}{5}+\frac{1}{3}\right)=2\cdot\frac{30-3-5}{15}=\frac{44}{15}$

(6) $y=x^2,\ y=2-x,\ y=0$, 1st quad. $(x=0)$

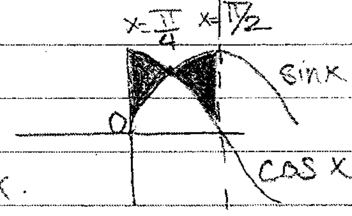


$x^2=2-x\Rightarrow x^2+x-2=0\Rightarrow(x+2)(x-1)\Rightarrow x=1$

$A={}_0\int^{1}2-x-x^2\,dx$

$A=2x-\frac{1}{2}x^2-\frac{1}{3}x^3\,\Big|_0^1=2-\frac{1}{2}-\frac{1}{3}=\frac{12-3-2}{6}=\frac{7}{6}$

(8) $y=\sin x,\ y=\cos x,\ x=0,\ y=\frac{\pi}{2}$



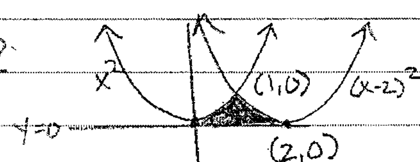
on $[0,\frac{\pi}{4}]$ $\cos x\ge\sin x$, $[\frac{\pi}{4},\frac{\pi}{2}]$ $\sin x\ge\cos x$

$A={}_0\int^{\pi/4}(\cos x-\sin x)\,dx+{}_{\frac{\pi}{4}}\int^{\frac{\pi}{2}}(\sin x-\cos x)\,dx$

$A=\sin x+\cos x\,\Big|_0^{\frac{\pi}{4}}+-\cos x-\sin x\,\Big|_{\frac{\pi}{4}}^{\frac{\pi}{2}}$

$A=\frac{\sqrt2}{2}+\frac{\sqrt2}{2}-(0+1)-0-1-\left(-\frac{\sqrt2}{2}-\frac{\sqrt2}{2}\right)=2\sqrt2-2$

(10) $y=x^2,\ y=(x-2)^2,\ y=0$ from $x=0$ to $x=2$

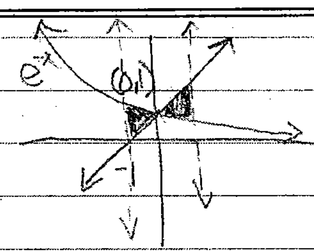


$x^2=(x-2)^2\Rightarrow x^2=x^2-4x+4\Rightarrow x=1$

$A={}_0\int^{1}x^2\,dx+{}_1\int^{2}(x^2-4x+4)\,dx$

$A=\frac{1}{3}x^3\,\Big|_0^1+\frac{1}{3}x^3-2x^2+4x\,\Big|_1^2=\frac{1}{3}+\frac{8}{3}-8+8-\left(\frac{1}{3}-2+4\right)=\frac{2}{3}$

(12) $y=e^{-x},\ y=x+1$ from $x=-1$ to $x=1$



$A={}_{-1}\int^{0}e^{-x}-(x+1)\,dx+{}_0\int^{1}x+1-e^{-x}\,dx$

$A=-e^{-x}-\frac{1}{2}x^2-x\,\Big|_{-1}^{0}+\frac{1}{2}x^2+x+e^{-x}\,\Big|_0^1$

$A=-1-\left(-e-\frac{1}{2}+1\right)+\frac{1}{2}+1+e^{-1}-1$

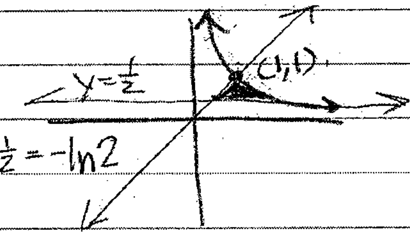
$A=-2+e+\frac{1}{2}+\frac{3}{2}+\frac{1}{e}-1=e+\frac{1}{e}-1$

(14) $x=y,\ x=\frac{1}{y},\ y=\frac{1}{2}$, 1st quad

$y=\frac{1}{y} \Rightarrow y^2=1 \Rightarrow y=\pm 1$

$A={}_{\frac{1}{2}}\int^{1} \frac{1}{y}-y\,dy = \ln|y|-\frac{1}{2}y^2\Big|_{1/2}^{1}$ $\ln\frac{1}{2}=-\ln 2$

$A=-\frac{1}{2}+\ln 2+\frac{1}{8}=\ln 2-\frac{3}{8}$



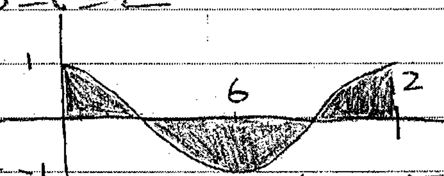
(16) Since $(y-1)^2+1 \geq (y-1)^2-1$ on $[0,2]$,

$A={}_0\int^2 (y-1)^2+1-((y-1)^2-1)\,dy = {}_0\int^2 2\,dy = 2y\Big|_0^2 = 4$

(18) $\frac{d}{dt}B(t)=\cos(\frac{\pi}{6}t)$ for $0\leq t\leq 12$

a) Graph $\frac{dB}{dt}$

Period $=\frac{2\pi}{\frac{\pi}{6}}=2\pi\cdot\frac{6}{\pi}=12$



b) $B(0)=B_0$ C.C. $=\int\cos(\frac{\pi}{6}u)\,du=\frac{6}{\pi}\sin(\frac{\pi}{6}t)$

and $B(t)=B(0)+$ C.C. $=B_0+\frac{6}{\pi}\sin(\frac{\pi}{6}t)$

$B(12)=B_0=B(0)$, C.C. $={}_0\int^{12}\cos(\frac{\pi}{6}t)\,dt$

Geometrically, $A^+=A^-$ ← area below $=B(12)-B(0)=0$

^ area above x-axis

(22) $\frac{dw}{dx}$ = rate of change of wt. at age x

${}_3\int^5 \frac{dw}{dx}\,dx$ is the cumulative (total) change in weight from age 3 to age 5

(24) $\frac{dN}{dt}=f(t)$, $N(t)$ = size of population at time t

${}_2\int^4 f(t)\,dt$

(26) $g(t)=e^{-t}$. Find ave. value over $[-1,1]$

$f_{ave}=\frac{1}{b-a}{}_a\int^b f(x)\,dx$ $g(t)$ is cont. on $[-1,1]$

$g_{ave}=\frac{1}{1-(-1)}{}_{-1}\int^1 e^{-t}\,dt=\frac{1}{2}(-e^{-t})\Big|_{-1}^{1}=\frac{1}{2}(-e^{-1}+e^1)=\frac{1}{2}(e-\frac{1}{e})$

(28) $y=673.8-34.7x$ for $0\leq x\leq 10$

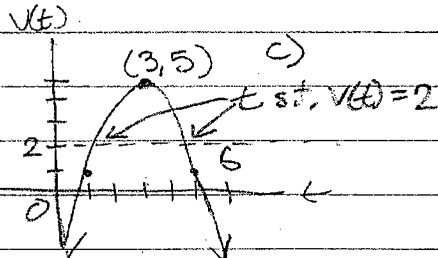
Ave. concent. $=\frac{1}{10-0}{}_0\int^{10} 673.8-34.7x\,dx$

$=\frac{1}{10}(673.8x-\frac{34.7}{2}x^2)\Big|_0^{10}$

$=\frac{1}{10}(6738-\frac{3470}{2})=\frac{1}{10}(5003)=500.3$

(32) $v(t) = -(t-3)^2 + 5$ for $0 \le t \le 6$

a) Graph $v(t)$ for $0 \le t \le 6$



b) Ave. Velocity $= \frac{1}{6-0} \int_0^6 -(t-3)^2+5 \, dt$

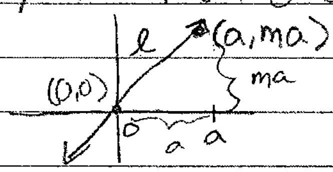
$V_{ave} = \frac{1}{6} \int_0^6 -t^2+6t-9+5 \, dt = \frac{1}{6}\left[-\frac{1}{3}t^3+3t^2-4t\right]_0^6$

$V_{ave} = \frac{1}{6}(-72+108-24) = \frac{12}{6} = 2$

c) $-(t^*-3)^2 = -3 \Rightarrow t^* = 3 \pm \sqrt{3}$

(54) Find length of $y=mx$ from 0 to a, $m,a>0$, using:

a) geometry



pythag. theorem:

$l^2 = a^2 + (ma)^2$

$l = \sqrt{a^2(1+m^2)} = a\sqrt{1+m^2}$

b) using formula

$L = \int_a^0 \sqrt{1+[f'(x)]^2} \, dx$ $L = \int_0^a \sqrt{1+m^2} \, dx = \sqrt{1+m^2} \cdot x \Big|_0^a$

$f(x) = mx \Rightarrow f'(x) = m$ $L = a\sqrt{1+m^2}$

(56) Find length of $2y^2 = 3x^3$ from 0 to 1

$y^2 = \frac{3}{2}x^3 \Rightarrow y = \pm\sqrt{\frac{3}{2}}\, x^{\frac{3}{2}} = f(x)$ $f'(x) = \pm\frac{3}{2}\sqrt{\frac{3}{2}}\, x^{\frac{1}{2}} \Rightarrow (f'(x))^2 = \frac{9}{4}\cdot\frac{3}{2}x$

$L = \int_0^1 \sqrt{1+\frac{27}{8}x} \, dx = \int_0^1 \left(1+\frac{27}{8}x\right)^{\frac{1}{2}} dx$

$\boxed{u = 1+\frac{27}{8}x \Rightarrow du = \frac{27}{8}dx \quad u(1) = \frac{8}{8}+\frac{27}{8} = \frac{35}{8},\ u(0) = 1}$

$L = \frac{8}{27}\int_1^{\frac{35}{8}} u^{\frac{1}{2}} du = \frac{8}{27}\cdot\frac{2}{3}u^{\frac{3}{2}} \Big|_1^{\frac{35}{8}}$

$L = \frac{16}{81}\left(\sqrt{\left(\frac{35}{8}\right)^3} - 1\right) \approx 1.61$

(58) $y = \frac{x^4}{4} + \frac{1}{8}x^{-2}$ from 2 to 4 | $y' = x^3 - \frac{1}{4}x^{-3} \Rightarrow (y')^2 = x^6 - 2\left(\frac{1}{4}\right)x^0 + \frac{1}{16}x^{-6}$

$(y')^2 + 1 = x^6 + \frac{2}{4}x^0 + \frac{1}{16}x^{-6} = \left(x^3 + \frac{1}{4}x^{-3}\right)^2$

$L = \int_2^4 x^3 + \frac{1}{4}x^{-3} \, dx$

$L = \frac{1}{4}x^4 - \frac{1}{8}x^{-2} \Big|_2^4 = 64 - \frac{1}{8\cdot 16} - \left(4 - \frac{1}{32}\right) = 60 - \frac{1}{128} + \frac{4}{128} = 60\frac{3}{128}$

(60) $y = \sin x$, $0 \le x \le \frac{\pi}{2}$ set up length

$y' = \cos x \Rightarrow (y')^2 = \cos^2 x$

$L = \int_0^{\frac{\pi}{2}} \sqrt{1+\cos^2 x} \, dx$

(62) $y = \ln x$, $1 \le x \le e$ $y' = \frac{1}{x} \Rightarrow (y')^2 = \frac{1}{x^2} \Rightarrow (y')^2 + 1 = \frac{x^2+1}{x^2}$

$L = \int_1^e \sqrt{\frac{x^2+1}{x^2}} \, dx$

$L = \int_1^e \frac{1}{x}\sqrt{x^2+1} \, dx$