Solutions to HW # 3

2. $\frac{dy}{dx} = 1 - x^3$

4. $\frac{dy}{dx} = 1 + x^4$

6. $\frac{dy}{dx} = \sqrt{2 + x^2}$

8. $\frac{dy}{dx} = \sqrt{\tan^2 x + 2}$

10. $\frac{dy}{dx} = x e^{-x^2}$

16. $h' = 2$ $g' = 0$, $\frac{dy}{dx} = ((2x-1)^2 - 1) \cdot 2 = 8x^2 - 8x$

18. $h' = 3$ $g' = 0$, $\frac{dy}{dx} = (1 + (3x+2)^3) \cdot 3 = 81x^3 + 162x^2 + 108x + 27$

20. $h' = 2x$ $g' = 0$, $\frac{dy}{dx} = (\sqrt{3 + (x^2 - 2)}) \cdot 2x = 2x\sqrt{x^2+1}$

22. $h' = 4x$ $g' = 0$ $\frac{dy}{dx} = (e^{-2(2x^2-1)} + 2) \cdot 4x = (e^{-4x^2+2} + 2) \cdot 4x$

24. $h' = \frac{1}{x}$ $g' = 0$. $\frac{dy}{dx} = (e^{-\ln x}) \cdot \frac{1}{x} = e^{\ln x^{-1}} \cdot \frac{1}{x} = \frac{1}{x} \cdot \frac{1}{x} = \frac{1}{x^2}$

26. $h' = 0$ $g' = 1$ $\frac{dy}{dx} = -(1 + e^x)$

28. $h' = 0$ $g' = 4x$ $\frac{dy}{dx} = -(1 + (2x^2)^2) \cdot 4x = -16x^5 - 4x$

30. $h' = 0$ $g' = 2x$ $\frac{dy}{dx} = -\left(\frac{1}{1+x^2}\right) \cdot 2x = \frac{-2x}{1+x^2}$

32. $h' = 0$ $g' = 2x$ $\frac{dy}{dx} = -\tan(1+x^2) \cdot 2x$

34. $h' = 1$ $g' = -1$ $\frac{dy}{dx} = \tan(x) - \tan(-x) \cdot (-1) = \tan(x) + \tan(-x) = 0$

36. $h' = 4x^3$ $g' = 3x^2$ $\frac{dy}{dx} = \ln(1 + x^4) \cdot 4x^3 - \ln(1 + x^3) \cdot 3x^2$

38. $h' = 3x^2 - 2$ $g' = 2x$ $\frac{dy}{dx} = \cos(x^3 - 2x) \cdot (3x^2 - 2) - \cos(1 + x^2) \cdot (2x)$

44. $\int \frac{1}{2}x^5 + 2x^3 - 1\,dx = \frac{1}{12}x^6 + \frac{1}{2}x^4 - x + C$

46. $\int \frac{x^2+2x}{2\sqrt{x}}\,dx = \int \frac{x^2}{2\sqrt{x}} + \frac{2x}{2\sqrt{x}}\,dx = \int \frac{1}{2}x^{3/2} + x^{1/2}\,dx = \frac{1}{5}x^{5/2} + \frac{2}{3}x^{3/2} + C$

48. $\int (1+x^3)\sqrt{x}\,dx = \int x^{\frac{1}{2}} + x^{7/2}\,dx = \frac{2}{3}x^{\frac{3}{2}} + \frac{2}{9}x^{9/2} + C$

50. $\int x^{3/5} + x^{5/3}\,dx = \frac{5}{8}x^{8/5} + \frac{3}{8}x^{8/3} + C$

52. $\int 2\sqrt{x} + \frac{1}{2\sqrt{x}}\,dx = \int 2x^{\frac{1}{2}} + \frac{1}{2}x^{-\frac{1}{2}}\,dx = \frac{4}{3}x^{3/2} + x^{\frac{1}{2}} + C$

54. $\int (x-1)^2\,dx = \int x^2 - 2x + 1\,dx = \frac{1}{3}x^3 - x^2 + x + C$

56. $\int (2x+3)^2\,dx = \int 4x^2 - 12x + 9\,dx = \frac{4}{3}x^3 - 6x^2 + 9x + C$

58. $\int 2e^{3x}\,dx = 2\int e^{3x}\,dx = \frac{2}{3}e^{3x} + C$

60. $\int 2e^{-x/3}\,dx = 2\int e^{-\frac{1}{3}x}\,dx = -6e^{-\frac{1}{3}x} + C$

62. $\int e^x(1-e^{-x})\,dx = \int e^x - e^x e^{-x}\,dx = \int e^x - 1 = e^x - x + C$

64. $\int \sin(1-x)\,dx = -\cos(1-x)\cdot(-1) + C = \cos(1-x) + C$

100. $\int_1^2 x^{5/2}\,dx = \frac{2}{7}x^{7/2}\Big|_1^2 = \frac{2}{7}(2)^{7/2} - \frac{2}{7}(1)^{\frac{7}{2}} = \frac{16}{7}\sqrt{2} - \frac{2}{7}$

102. $\int_4^9 \frac{1+\sqrt{x}}{\sqrt{x}}\,dx = \int_4^9 \frac{1}{\sqrt{x}} + 1\,dx = \int_4^9 x^{-\frac{1}{2}} + 1\,dx = 2x^{\frac{1}{2}} + x\Big|_4^9$

$= 2(9)^{\frac{1}{2}} + (9) - (2(4)^{\frac{1}{2}} + (4)) = 7$

104. $\int_{-1}^2 (4-3t)^2\,dt = \int_{-1}^2 9t^2 - 24t + 16\,dt = 3t^3 - 12t^2 + 16t\Big|_{-1}^2$

$= 3(2)^3 - 12(2)^2 + 16(2) - (3(-1)^3 - 12(-1)^2 + 16(-1))$

$= 39$

106. $\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} 2\cos\left(\frac{x}{2}\right)dx = 2\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \cos\left(\frac{x}{2}\right)dx = 2\cdot 2\sin\frac{x}{2}\Big|_{-\frac{\pi}{3}}^{\frac{\pi}{3}}$

$= 4\sin\frac{\pi}{6} - 4\sin\frac{-\pi}{6} = 4\left(\frac{1}{2}\right) - 4\left(-\frac{1}{2}\right) = 4$

108 $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \tan x\,dx = \ln|\sec x|\Big]_{-\frac{\pi}{4}}^{\frac{\pi}{4}} = \ln\left|\sec\frac{\pi}{4}\right| - \ln\left|\sec\frac{-\pi}{4}\right|$

$= \ln|\sqrt{2}| - \ln|\sqrt{2}| = 0$

110. $\int_{-\sqrt{3}}^{-1} \frac{4}{1+x^2}dx = 4\int_{-\sqrt{3}}^{-1} \frac{1}{1+x^2}dx = 4\tan^{-1}x\Big|_{-\sqrt{3}}^{-1} = 4\tan^{-1}(-1) - 4\tan^{-1}(-\sqrt{3})$

$= 4\left(-\frac{\pi}{4}\right) - 4\left(-\frac{\pi}{3}\right) = \frac{\pi}{3}$

112. $\int_{-1}^{1} \frac{3}{\sqrt{1-x^2}}dx = 3\int_{-1}^{1} \frac{1}{\sqrt{1-x^2}}dx = 3\sin^{-1}x\Big|_{-1}^{1} = 3\sin^{-1}(1) - 3\sin^{-1}(-1)$

$= 3\left(\frac{\pi}{2}\right) - 3\left(-\frac{\pi}{2}\right) = 3\pi$

114. $\int_{\frac{\pi}{20}}^{\frac{\pi}{15}} \sec(5x)\tan(5x)\,dx = \frac{1}{5}\sec(5x)\Big|_{\frac{\pi}{20}}^{\frac{\pi}{15}} = \frac{1}{5}\sec\left(5\cdot\frac{\pi}{15}\right) - \frac{1}{5}\sec\left(5\cdot\frac{\pi}{20}\right)$

$= \frac{1}{5}\sec\left(\frac{\pi}{3}\right) - \frac{1}{5}\sec\left(\frac{\pi}{4}\right) = \frac{1}{5}\cdot 2 - \frac{1}{5}\sqrt{2} = \frac{2}{5} - \frac{\sqrt{2}}{5}$

116. $\int_0^2 2te^{t^2}dt = e^{t^2}\Big|_0^2 = e^{(2)^2} - e^{(0)^2} = e^4 - 1$

118. $\int_{-1}^{1} e^{-|s|}ds = \int_{-1}^{0} e^{-|s|}ds + \int_0^1 e^{-|s|}ds = \int_{-1}^{0} e^{-(-s)}ds + \int_0^1 e^{-s}ds$

$= \int_{-1}^{0} e^{s}ds + \int_0^1 e^{-s}ds = e^{s}\Big|_{-1}^{0} + -e^{-s}\Big|_0^1$

$= e^0 - e^{-1} + (-e^{-1} + e^0) = 2 - 2e^{-1}$

120 $\int_2^3 \frac{1}{z+1}dz = \ln(z+1)\Big|_2^3 = \ln(3+1) - \ln(2+1)$

$= \ln(4) - \ln(3) = \ln\left(\frac{4}{3}\right)$