Homework 2 p355

4/ We divide $[0,1]$ into 5 equal subintervals, $[0,0.2]$, $[0.2,0.4]$, $[0.4,0.6]$, $[0.6,0.8]$, $[0.8,1]$
(This corresponds to the partition P given by: $a=0<0.2<0.4<0.6<0.8<1=b$)
For each subinterval we have $\Delta x_K = 0.2$ so

$$S_P = \sum_{K=1}^{5} f(c_K)\Delta x_K \quad \text{where } f = 1-x^2$$

a) Left end points: $c_1=0$, $c_2=0.2$, $c_3=0.4$, $c_4=0.6$, $c_5=0.8$

so
$$\begin{aligned} S_P &= \sum_{K=1}^{5} f(c_K)\Delta x_K \\ &= \sum_{K=1}^{5} (1-c_K^2)(0.2) \\ &= 0.2 \sum_{K=1}^{5} (1-c_K^2) \\ &= 0.2\left[(1-0^2)+(1-(0.2)^2)+(1-(0.4)^2)+(1-(0.6)^2)+(1-0.8^2)\right] \\ &= 0.2 \times [4.8] = \underline{0.96} \end{aligned}$$

b) Right end points: $c_1=0.2$, $c_2=0.4$, $c_3=0.6$, $c_4=0.8$, $c_5=1$

$$\begin{aligned} S_P &= \sum_{K=1}^{5} (1-c_K^2)(0.2) \quad \text{(as above)} \\ &= 0.2\left[(1-0.2^2)+(1-0.4^2)+\dots+(1-0.8^2)+(1-1^2)\right] \\ &= 0.2[3.8] = \underline{0.76} \end{aligned}$$

GRADED

$\boxed{6}$ $\sum_{K=3}^{6} K^2 = 3^2 + 4^2 + 5^2 + 6^2$

8 $\sum_{K=1}^{3} \frac{K}{K+1} = \frac{1}{1+1} + \frac{2}{2+1} + \frac{3}{3+1} = \frac{1}{2} + \frac{2}{3} + \frac{3}{4}$

10 $\sum_{K=0}^{4} K^x = 0^x + 1^x + 2^x + 3^x + 4^x = 1 + 1^x + 2^x + 3^x + 4^x$

12 $\sum_{K=1}^{n} f(c_K)\Delta x_K = f(c_1)\Delta x_1 + f(c_2)\Delta x_2 + \dots + f(c_{n-1})\Delta x_{n-1} + f(c_n)\Delta x_n$

14 $\sum_{K=1}^{n} \cos\left(K\frac{\pi}{n}\right)\frac{\pi}{n} = \cos\left(\frac{\pi}{n}\right)\frac{\pi}{n} + \cos\left(\frac{2\pi}{n}\right)\frac{\pi}{n} + \dots + \cos\left(\frac{(n-1)\pi}{n}\right)\frac{\pi}{n} + \cos\left(\frac{n\pi}{n}\right)\frac{\pi}{n}$

$= \frac{\pi}{n}\left(\cos\left(\frac{\pi}{n}\right) + \cos\left(\frac{2\pi}{n}\right) + \dots + \cos\left(\frac{(n-1)\pi}{n}\right) - 1\right)$

GRADED

$\boxed{16}$ $\sum_{K=1}^{4} \frac{1}{\sqrt{K}}$

18 $\sum_{K=3}^{7} \frac{K}{K+2}$ $\left(\text{or } \sum_{K=5}^{9} \frac{K-2}{K} \text{ etc...}\right)$

20 $\sum_{K=0}^{n} \frac{1}{2^K}$ $\left(\text{or } \sum_{K=1}^{n+1} \frac{1}{2^{K-1}} \text{ etc...}\right)$

22 $\sum_{K=0}^{n} (-1)^K a^K$

$\boxed{24}$ $\sum_{K=1}^{10} (2-K^2) = \sum_{K=1}^{10} 2 - \sum_{K=1}^{10} K^2$

GRADED

$= 2\left(\sum_{K=1}^{10} 1\right) - \left(\sum_{K=1}^{10} K^2\right)$

$= 2(10) - \left(\frac{10(10+1)(20+1)}{6}\right)$

$= 20 - 385 = \underline{-365}$

use formula for $\sum_{K=1}^{n} a^2$ in book!

26 $\sum_{K=1}^{n} 3K = 3\sum_{K=1}^{n} K = 3\frac{n(n+1)}{2} = \frac{3n(n+1)}{2}$

28. $\sum_{k=1}^{n} (k+2)(k-2) = \sum_{k=1}^{n} k^2 - 4$

$= \sum_{k=1}^{n} k^2 - \sum_{k=1}^{n} 4$

$= \frac{n(n+1)(2n+1)}{6} - 4n$

GRADED

$\boxed{30}$ $\sum_{k=1}^{11} (-1)^k = \underbrace{\left((-1) + (1) + (-1) + \ldots + (1) + (-1)\right)}_{\text{there are eleven of these,}}$

$= -1$

32. $\int_{-1}^{1} (1-x^2)\,dx$

(Our partition will be $P = [-1, -\frac{3}{5}, -\frac{1}{5}, \frac{1}{5}, \frac{3}{5}, 1]$

(Our subintervals are $[-1, -\frac{3}{5}], [-\frac{3}{5}, -\frac{1}{5}], [-\frac{1}{5}, \frac{1}{5}], [\frac{1}{5}, \frac{3}{5}], [\frac{3}{5}, 1]$)

So in each case $\Delta x_k = \frac{2}{5}$, and with left endpoints

$S_p = \sum_{k=1}^{5} (1 - c_k^2)\,\Delta x_k$

$= \frac{2}{5} \sum_{k=1}^{5} (1 - c_k^2)$

$= \frac{2}{5}\left[\left(1-(-1)^2\right) + \left(1-(-\frac{3}{5})^2\right) + \left(1-(-\frac{1}{5})^2\right) + \left(1-(\frac{1}{5})^2\right) + \left(1-(\frac{3}{5})^2\right)\right]$

$= \frac{2}{5}\left[5 - \frac{9}{5}\right] = \frac{32}{25}$

34. As above $S_p = \frac{2}{5}\left[\left(1-(-\frac{3}{5})^2\right) + \left(1-(-\frac{1}{5})^2\right) + \left(1-(\frac{1}{5})^2\right) + \left(1-(\frac{3}{5})^2\right) + \left(1-1^2\right)\right]$

$= \frac{2}{3}\left(5 - \frac{4}{5}\right) = \frac{2}{5}\left(\frac{21}{5}\right) = \frac{42}{25}$

GRADED

$\boxed{36}$ $\int_{-1}^{2} e^{-x}\,dx$

3 equal sub-intervals are $[-1,0], [0,1], [1,2]$
3 mid points are $c_1 = -\frac{1}{2}$, $c_2 = \frac{1}{2}$, $c_3 = \frac{3}{2}$

So $S_p = \sum_{K=1}^{3} f(c_K)\Delta x_K$

$= \sum_{K=1}^{3} f(c_K)$ $(\Delta x_K = 1)$

$= e^{-c_1} + e^{-c_2} + e^{-c_3}$

$= e^{-(-\frac{1}{2})} + e^{-(\frac{1}{2})} + e^{-(\frac{3}{2})}$

$= 6.74$

40 (Not sure about 'geometric argument'(?) but this works :)

By example 1 we know $\int_0^a x^2\,dx = \frac{a^3}{3}$, for any $a \in \mathbb{R}$.
(We know x^2 is continuous on all of $\mathbb{R}$ so by page 344, $\int_a^b x^2\,dx$ exists for any $a,b \in \mathbb{R}$).

So by properties 2) then 5) p349 + (and example 1) we have:

$$\int_a^b x^2\,dx = \int_a^0 x^2\,dx + \int_0^b x^2\,dx$$
$$= \int_0^b x^2\,dx - \int_0^a x^2\,dx$$
$$= \frac{b^3}{3} - \frac{a^3}{3} = \frac{b^3 - a^3}{3}.$$

42 $\int_1^4 \sqrt{x}\,dx$

$\boxed{44}$ $\int_1^2 \frac{1}{x+1}\,dx$

GRADED

46 $\int_0^{\pi} \sin x \, dx$

48 $\int_{-2}^{-1} \frac{x}{1-x}\,dx = \lim_{\|P\|\to 0} \sum_{k=1}^{n} \frac{c_k}{1-c_k}\Delta x_k$ where P is a partition of $[-2,-1]$

GRADED

$\boxed{50}$ $\int_1^3 e^{-x}\,dx = \lim_{\|P\|\to 0} \sum_{k=1}^{n} e^{-c_k}\Delta x_k$ " P " " " " $[1,3]$

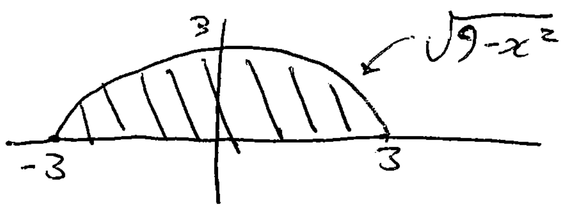
52 $\int_0^{\pi} \cos(2x)\,dx = \lim_{\|P\|\to 0} \sum_{k=1}^{n} \cos(2c_k)\Delta x_k$ " " " " " $[0,\pi]$

54 $\int_{-2}^{3} (x-3)\,dx = \lim_{\|P\|\to 0} \sum_{k=1}^{n} \cancel{\cos}(c_k-3)\Delta x_k$ " " " " " $[-2,3]$

GRADED

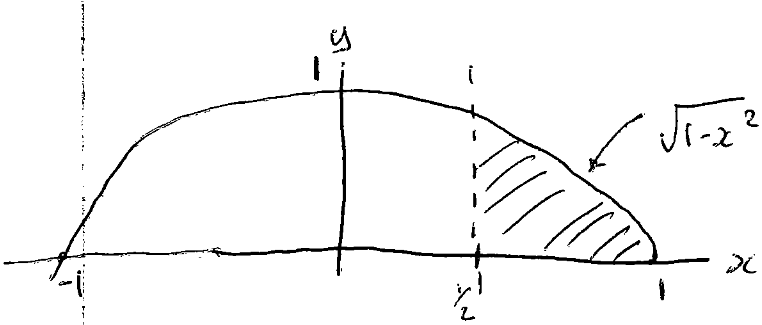
$\boxed{62}$ Note that $f(x)=\sqrt{9-x^2}$ is half circle in the upper half of the plane (see below), of radius 3.

Thus $\int_{-3}^{3}\sqrt{9-x^2}\,dx$ is the area bounded by this semi circle and the ~~ax~~ x axis (shaded area below:)



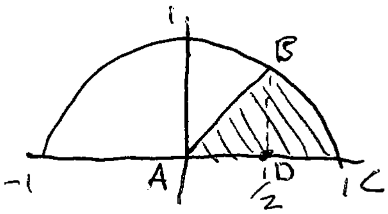
GRADED So $\int_{-3}^{3}\sqrt{9-x^2}\,dx = \frac{1}{2}(\pi r^2) = \frac{1}{2}(\pi \times 9) = \frac{9\pi}{2}$

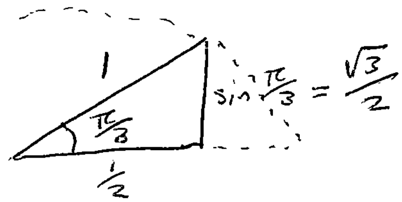
$\boxed{64}$ As in question 62, except radius is now 1 and we want the area bounded by the ~~semisphere~~, semicircle, the x-axis and the line $x=\frac{1}{2}$:



(\\\ shaded area $= \int_{\frac{1}{2}}^{1}\sqrt{1-x^2}\,dx$)

We begin by finding this area:





We note that since the x-coordinate of B is $\frac{1}{2}$, the angle AB makes with the x-axis is $\frac{\pi}{3}$ (since $\cos(\frac{\pi}{3}) = \frac{1}{2}$).

Therefore the shaded region is $\frac{(\frac{\pi}{3})}{2\pi}$ of the area of the whole circle, ie

$$\text{shaded area} = \frac{(\frac{\pi}{3})}{2\pi} \times (\pi(1)^2) = \frac{\pi}{6}$$

Then to find the required area we subtract the area of the triangle ABD from the above area. This gives:

$$\int_{\frac{1}{2}}^{1} \sqrt{1-x^2}\,dx = \frac{\pi}{6} - \left(\frac{1}{2} \times b \times h\right)$$
$$= \frac{\pi}{6} - \left(\frac{1}{2} \times \frac{1}{2} \times \frac{\sqrt{3}}{2}\right)$$
$$= \frac{\pi}{6} - \frac{\sqrt{3}}{8}$$

68 a) $\int_0^2 \frac{1}{2}x^2\,dx = \frac{1}{2}\int_0^2 x^2\,dx$ (p349, prop. 3)

$= \frac{1}{2}\left(\frac{1}{3}(2)^3\right)$ (info in question)

$= \frac{4}{3}$

68

b) $\int_{-3}^{0} 2x^2\,dx = -\int_{0}^{-3} 2x^2\,dx$ (p 349, prop 2)

$= -2\int_{0}^{-3} x^2\,dx$ (p 349, prop 3)

$= -2\left(\frac{1}{3}(-3)^3\right)$ (info in question)

$= 18$

GRADED

$\boxed{c)}$ $\int_{1}^{3} \frac{1}{3}x^2\,dx = \frac{1}{3}\int_{1}^{3} x^2\,dx$ (property 3)

$= \frac{1}{3}\left[\int_{1}^{0} x^2\,dx + \int_{0}^{3} x^2\,dx\right]$ (property 5)

$= \frac{1}{3}\left[\int_{0}^{3} x^2\,dx - \int_{0}^{1} x^2\,dx\right]$ (property 2)

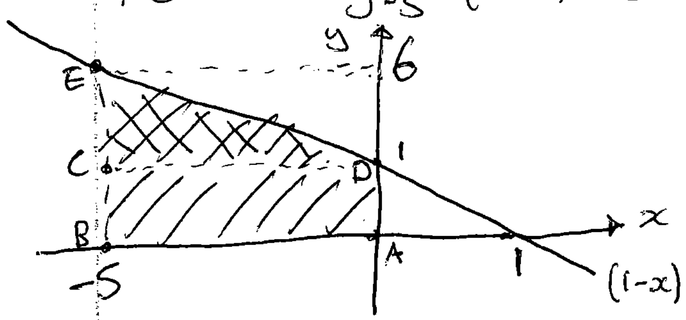
$= \frac{1}{3}\left[\frac{(3)^3}{3} - \frac{(1)^3}{3}\right]$ (info in quest.)

$= \underline{\underline{\frac{26}{9}}}$

72

$\int_{0}^{-5} (1-x)\,dx = -\int_{-5}^{0} (1-x)\,dx$ (property 2)

The area $\int_{-5}^{0} (1-x)\,dx$ is shown below (shaded):



Area = Area of triangle EDC

+ Area of rectangle BCDA

$= \frac{1}{2}\times(5)\times(5) + 5\times 1$

$= \frac{25}{2} + 5 = \frac{35}{2}$

So $\int_{0}^{-5}(1-x)\,dx = -\left(\int_{-5}^{0}(1-x)\,dx\right) = -\left(\frac{35}{2}\right) = \underline{\underline{-\frac{35}{2}}}$.