HW #10

(20) $\frac{dW}{dt} = -\lambda W(t)$, $W(0) = W_0$ & $\lambda > 0$

a) Find $W(t)$

$\frac{dW}{-\lambda W} = dt \Rightarrow t = -\frac{1}{\lambda}\int \frac{1}{W}\,dW = -\frac{1}{\lambda}\ln|W| + c$

$\Rightarrow -\lambda t + \lambda c = \ln|W| \Rightarrow \pm e^{-\lambda t} \cdot e^{\lambda c} = W$ (Let $C_0 = \pm e^{\lambda c}$)

So $W(t) = C_0 e^{-\lambda t} \Rightarrow W_0 = C_0 e^{-\lambda \cdot 0} = C_0 \Rightarrow W(t) = W_0 e^{-\lambda t}$

b) $W_0 = W(0) = 123$ gr $\Rightarrow W(t) = 123 e^{-\lambda t}$

so $W(5) = 20$gr $\Rightarrow 20 = 123 e^{-5\lambda} \Rightarrow \frac{20}{123} = e^{-5\lambda}$

$\Rightarrow \ln\left(\frac{20}{123}\right) = -5\lambda \Rightarrow \lambda = -\frac{1}{5}\ln\left(\frac{20}{123}\right) \approx .36329$

½ life: t so that $\frac{1}{2} = e^{\frac{1}{5}\ln\left(\frac{20}{123}\right)t} \Rightarrow -\ln 2 = \frac{1}{5}\ln\left(\frac{20}{123}\right)t$

$\Rightarrow t = \frac{-5\ln 2}{\ln\left(\frac{20}{123}\right)} \approx 1.9$ minutes

(22) $\frac{dL}{dt} = k(34 - L(t))$, $L(0) = 2$

a) $\frac{dL}{34-L} = k\,dt \Rightarrow kt = \int \frac{dL}{34-L} = -\ln|34 - L| + c$

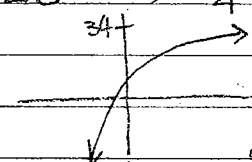
$\Rightarrow -kt + c = \ln|34 - L| \Rightarrow e^{-kt} e^{c} = |34 - L|$ Let $C_0 = \pm e^{c}$

so $e^{-kt} \cdot C_0 = 34 - L \Rightarrow L = 34 - C_0 e^{-kt} \Rightarrow 2 = 34 - C_0$

so $L(t) = 34 - 32e^{-kt}$

b) $L(4) = 10$ so $10 = 34 - 32e^{-4k} \Rightarrow \frac{3}{4} = e^{-4k} \Rightarrow -4k = \ln\left(\frac{3}{4}\right)$

$\Rightarrow k = -\frac{1}{4}\ln\left(\frac{3}{4}\right) \approx .0719...$



c) $L(10) = 34 - 32e^{\frac{1}{4}\ln\left(\frac{3}{4}\right)\cdot 10} = 34 - 32 \cdot \left(\frac{3}{4}\right)^{\frac{5}{2}} \approx 18.412$

d) $\lim_{t\to\infty}\left(34 - 32e^{-kt}\right) = 34 - 32 \cdot 0 = 34$ $(k > 0)$

(26) $\frac{dy}{dx} = y(1-y)$ $(0,2)$

$\frac{dy}{y(1-y)} = dx \Rightarrow x = \int \frac{dy}{y(1-y)} = \int \frac{a}{y}dy + \int \frac{b}{1-y}dy$

$$\boxed{\begin{array}{c} 1 = a(1-y) + by \\ y=1:\ b=1 \quad y=0:\ a=1 \end{array}}$$

$\Rightarrow x = \int \frac{1}{y}dy + \int \frac{1}{1-y}dy = \ln|y| - \ln|1-y| + C$

$\Rightarrow x - c = \ln\left|\frac{y}{1-y}\right| \Rightarrow e^{x} \cdot e^{-c} = \left|\frac{y}{1-y}\right|$ (so let $C_0 = \pm e^{-c}$)

$\Rightarrow y = (1-y)e^{x} \cdot C_0 = e^{x}C_0 - ye^{x}C_0 \Rightarrow y + ye^{x}C_0 = e^{x}C_0$

So $y = \frac{e^{x}C_0}{1 + e^{x}C_0} \Rightarrow 2 = \frac{C_0}{1 + C_0} \Rightarrow 2 + 2C_0 = C_0 \Rightarrow C_0 = -2$

So $y = \frac{-2e^{x}}{1 - 2e^{x}}$

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p.491 2-16e, 20ab, 22abcd, 26-30e, 34, 36, 44, 46, 48

(2) $\frac{dy}{dx} = e^{-x}$ $(0,10)$ $\Rightarrow dy = e^{-x}dx \Rightarrow y = \int e^{-x}dx = -e^{-x} + C$

$10 = -e^{0} + C = -1 + C \Rightarrow C = 11$ so $y = -e^{-x} + 11$

(4) $\frac{dy}{dx} = \frac{1}{1+x^2}$ $(0,1) \Rightarrow dy = \frac{1}{1+x^2}dx \Rightarrow y = \int \frac{1}{1+x^2}dx = \tan^{-1}x + C$

$1 = \tan^{-1}(0) + C = 0 + C$ so $y = \tan^{-1}x + 1$

(6) $\frac{dx}{dt} = \cos(t-3)$, $x(3) = 1$

$dx = \cos(t-3)dt \Rightarrow x = \int \cos(t-3)dt = \sin(t-3) + C$

$1 = \sin(3-3) + C \Rightarrow 1 = C$ so $x(t) = \sin(t-3) + 1$

(8) $\frac{dh}{dt} = 5 - 16t^2$, $h(3) = -11$

$dh = (5-16t^2)dt \Rightarrow h = \int 5-16t^2\, dt = 5t - \frac{16}{3}t^3 + C$

$-11 = 5(3) - \frac{16}{3}(3)^3 + C \Rightarrow C = 118$ so $h(t) = 5t - \frac{16}{3}t^3 + 118$

(10) $\frac{dP}{dt} = 3t + 1$, $P(0) = 0$ $P(10) = ?$

$dP = (3t+1)dt \Rightarrow P = \int 3t+1\, dt = \frac{3}{2}t^2 + t + C$ & $P(0) = 0 \Rightarrow C = 0$

so $P(t) = \frac{3}{2}t^2 + t \Rightarrow P(10) = 150 + 10 = 160$

(12) $\frac{dy}{dx} = 2(1-y)$, $(0,2)$

$\frac{dy}{2(1-y)} = dx \Rightarrow x = \frac{1}{2}\int \frac{dy}{1-y} \Rightarrow x = -\frac{1}{2}\ln|1-y| + C \Rightarrow -2x + 2C = \ln|1-y|$

$\Rightarrow e^{-2x}C_0 = 1 - y$ (let $C_0 = \pm e^{2C}$) $\Rightarrow C_0 = 1 - 2 = -1$ so $y = 1 + e^{-2x}$

(14) $\frac{dx}{dt} = 1 - 3x$, $x(-1) = -2$

$\frac{dx}{1-3x} = dt \Rightarrow t = \int \frac{dx}{1-3x} \Rightarrow t = -\frac{1}{3}\ln|1-3x| + C \Rightarrow -3t + 3C = \ln|1-3x|$

$\Rightarrow e^{-3t+3C} = |1-3x| \Rightarrow e^{-3t}\cdot C_0 = 1 - 3x$ (where $C_0 = \pm e^{3C}$)

so $1 - 3x = e^{-3t}\cdot C_0 \Rightarrow x = \frac{1}{3} - \frac{1}{3}e^{-3t}C_0 \Rightarrow -2 = \frac{1}{3} - \frac{1}{3}e^{3}\cdot C_0$

$\Rightarrow -\frac{7}{3} = -\frac{1}{3}e^{3}C_0 \Rightarrow C_0 = 7e^{-3} \Rightarrow x(t) = \frac{1}{3} - \frac{7}{3}e^{-3t-3}$

(16) $\frac{dN}{dt} = 5 - \frac{1}{2}N$, $N(2) = 3$

$\frac{dN}{5-\frac{1}{2}N} = dt \Rightarrow t = \int \frac{1}{5-\frac{1}{2}N}dt = -2\ln|5-\frac{1}{2}N| + C \Rightarrow -\frac{1}{2}t + \frac{1}{2}C = \ln|5 - \frac{1}{2}N|$

$\Rightarrow e^{-\frac{1}{2}t}e^{\frac{1}{2}C} = |5 - \frac{1}{2}N| \Rightarrow e^{-\frac{1}{2}t}\cdot C_0 = 5 - \frac{1}{2}N$ (where $C_0 = \pm e^{\frac{1}{2}C}$)

so $\frac{1}{2}N = 5 - e^{-\frac{1}{2}t}\cdot C_0 \Rightarrow N = 10 - 2e^{-\frac{1}{2}t}\cdot C_0 \Rightarrow 3 = 10 - 2e^{-1}\cdot C_0 \Rightarrow C_0 = \frac{7e}{2}$

$\Rightarrow N(t) = 10 - 7e\cdot e^{-\frac{1}{2}t} = 10 - 7e^{1-\frac{1}{2}t}$

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(28) $\frac{dy}{dx} = (y-1)(y-2)$, $(0,0)$

$\boxed{1 = a(y-2) + b(y-1) \quad y=2: b=1,\ y=1: a=-1}$

$x = \int \frac{dy}{(y-1)(y-2)} = \int \frac{a}{y-1}dy + \int \frac{b}{y-2}dy$

$x = -\int \frac{1}{y-1}dy + \int \frac{1}{y-2}dy = -\ln|y-1| + \ln|y-2| + C = \ln\left|\frac{y-2}{y-1}\right| + C$

$\Rightarrow e^{x-C} = \left|\frac{y-2}{y-1}\right|$ (let $C_0 = \pm e^{-C}$) $C_0e^x = \frac{y-2}{y-1} \Rightarrow y-2 = C_0e^x y - C_0e^x$

$\Rightarrow y - C_0e^x y = 2 - C_0e^x \Rightarrow y = \frac{2-C_0e^x}{1-C_0e^x}$ $\quad (0,0) \Rightarrow C_0 = \frac{-2}{-1} = 2$

SO $y = \frac{2-2e^x}{1-2e^x}$

(30) $\frac{dy}{dt} = \frac{1}{2}y^2 - 2y \Rightarrow 2\frac{dy}{dt} = y^2 - 4y = y(y-4)$, $(0,-3)$

$\boxed{1 = a(y-4) + by \quad y=4: b=\frac{1}{4},\ y=0: a=-\frac{1}{4}}$

$t = 2\int \frac{dy}{y(y-4)} = 2\left(\int \frac{a}{y}dy + \int \frac{b}{y-4}dy\right)$

$= \frac{1}{2}\left(\int \frac{1}{y-4}dy - \int \frac{1}{y}dy\right)$

$\Rightarrow 2t = \ln|y-4| - \ln|y| = \ln\left|\frac{y-4}{y}\right| + C \Rightarrow e^{2t}e^{-C} = \left|\frac{y-4}{y}\right|$ (let $C_0 = \pm e^{-C}$)

$\Rightarrow C_0e^{2t} = \frac{y-4}{y} \Rightarrow 4 = y - yC_0e^{2t} = y(1-C_0e^{2t}) \Rightarrow y = \frac{4}{1-C_0e^{2t}}$

$(0,-3) \Rightarrow \frac{7}{3} = C_0$ SO $y = \frac{4}{1-\frac{7}{3}e^{2t}}$

(34) $\frac{dy}{dx} = (3-y)(2+y) \Rightarrow \frac{dy}{(3-y)(2+y)} = dx \Rightarrow x = \int \frac{a}{3-y}dy + \int \frac{b}{2+y}dy$

$x = \frac{1}{5}\left(\int \frac{1}{3-y}dy + \int \frac{1}{2+y}dy\right)$

$\boxed{1 = a(2+y) + b(3-y) \quad y=-2: b=\frac{1}{5},\ y=3: a=\frac{1}{5}}$

$5x = \ln|3-y| + \ln|2+y| + C \Rightarrow e^{5x-C} = |(3-y)(2+y)|$

let $C_0 = \pm e^{-C}$ so $C_0e^{5x} = 6 + y - y^2 \Rightarrow y^2 - y + (C_0e^{5x} - 6) = 0$

$y = \frac{1 \pm \sqrt{25 - 4C_0e^{5x}}}{2}$ by the quad formula

(36) $\frac{dy}{dx} = y^2 + 4$, $(0,2)$

$x = \int \frac{dy}{y^2+4} = \frac{1}{2}\tan^{-1}\left(\frac{y}{2}\right) + C$, $0 = \tan^{-1}(1) + C \Rightarrow C = 0$

$2x = \tan^{-1}\left(\frac{y}{2}\right) \Rightarrow \tan(2x) = \frac{y}{2} \Rightarrow y = 2\tan(2x)$

(44) $\frac{dy}{dx} = 2\frac{y}{x}$, $(1,1)$

$\frac{dy}{y} = 2\frac{dx}{x} \Rightarrow \ln|y| = 2\ln|x| + C \Rightarrow |y| = |x|^2 \cdot e^C$ (let $C_0 = \pm e^C$)

$y = C_0x^2 \Rightarrow C_0 = 1$ SO $y = x^2$ (Check: $\frac{dy}{dx} = 2x = 2\left(\frac{x^2}{x}\right) = 2\frac{y}{x}$)

(46) $\frac{dy}{y} = \frac{dx}{x+1}$; $(0,2)$

$\ln|y| = \ln|x+1| + C \Rightarrow |y| = |x+1| \cdot e^C$, (let $C_0 = \pm e^C$)

$y = (x+1) \cdot C_0 \Rightarrow 2 = (0+1)C_0 = C_0$

So $y = 2(x+1) = 2x+2$ (Check: $\frac{dy}{dx} = \frac{y}{x+1} = \frac{2(x+1)}{x+1} = 2$)

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(48) $\frac{dy}{dx} = x^2 y^2 \quad (1,1)$

$\int y^{-2} dy = \int x^2 dx$

$-y^{-1} = \frac{1}{3}x^3 + C \Rightarrow -1 = \frac{1}{3} + C \Rightarrow C = -\frac{4}{3}$

$\frac{1}{y} = -\frac{1}{3}x^3 + \frac{4}{3} \Rightarrow y = \frac{1}{\frac{4}{3} - \frac{x^3}{3}} = \frac{3}{4 - x^3}$

Check: $\frac{dy}{dx} = \frac{9}{(4-x^3)^2} \cdot x^2$ & quotient rule $\frac{dy}{dx} = \frac{-3(-3x^2)}{(4-x^3)^2}$ ✓