Math 11B
Final Review Problems

Note: these problems represent *some* of the topics that were covered since Midterm 2. Bear in mind that the final will be comprehensive, with a slight emphasis on the most recent topics. See earlier review problems and solutions to Midterms 1 and 2 for review of earlier topics.

1. Use Integration by parts to prove the following formula: $\int x^n e^{ax}\,dx = \frac{1}{a}x^n e^{ax} - \frac{n}{a}\int x^{n-1}e^{ax}\,dx$.

2. Find the linear approximation to $f(x)=e^x$ for x near 2.

3. Find the n^{th} degree Taylor polynomial of the following functions $f(x)$ for x near a.
   a. $f(x)=e^x$, $n=5$, $a=0$
   b. $f(x)=e^x$, $n=5$, $a=1$
   c. $f(x)=e^{-x^2}$, $n=10$, $a=0$ (Hint: substitute into your answer to part **<u>a</u>**) Note: in an earlier version of this document, the hint said part b, which was an error. Let me expand on the hint a little. Take your answer for part a (not b), and replace x everywhere you see it by $-x^2$. This is much easier than taking 10 derivatives of e^{-x^2}.
   d. $f(x)=\sin x$, $n=7$, $a=0$
   e. $f(x)=\sqrt{1+x}$, $n=4$, $a=0$
   f. $f(x)=\ln x$, $n=4$, $a=1$

4. Use your answer to problem 3*c* above to estimate the value of the definite integral $\int_0^1 e^{-x^2}\,dx$. (Note you cannot calculate this definite integral directly by the FTC, so don't try.)

5. Use the remainder term $R_{n+1}(x)=\frac{f^{(n+1)}(c)}{(n+1)!}x^{n+1}$ for the n^{th} degree Taylor polynomial to determine the smallest value of n which would guarantee that in estimating $f(x)=e^x$ for $x\in(-2,2)$, the error would not exceed 10^{-4}.

6. Solve the following initial value problems.
   a. $\frac{dy}{dx}=-2xy$, $y(1)=2$
   b. $\frac{dP}{dt}=Pe^{-t}$, $P(0)=1$
   c. $\frac{dP}{dt}=e^{-t}$, $P(0)=1$
   d. $\frac{dN}{dt}=(1+N^2)\cos(t)$, $N(0)=1$
   e. $\frac{dN}{dt}=1+N^2$, $N(0)=\sqrt{3}$
   f. $\frac{dy}{dx}=y(y-1)$, $y(0)=y_0$

7. Consider the autonomous differential equation $\frac{dy}{dx} = y^4 - y^2$
    a. Determine the equilibrium solutions
    b. Compute the eigenvalues of the equation
    c. Classify each of the equilibria as either stable, unstable, or semi-stable

Since I fixed the above typo, let me add one more problem.

8. Newton's law of cooling says that the rate of change of temperature of an object is proportional to the difference between the temperature of the object and the ambient temperature. To put it more succinctly: $\frac{dT}{dt} = k(T(t) - A)$, where $T(t)$ is the temperature of the object at time t, A is the ambient temperature and k is the constant of proportionality.
    a. Solve the initial value problem $\frac{dT}{dt} = k(T(t) - A)$, $T(0) = T_0$.
    b. Assume the ambient temperature A is 10, $T(0) = 20$, and $T(5) = 12$. Determine k.

And here's one more:

9. Problem 20 on p.491 in the text