7.6 TABLES OF INTEGRALS

EXERCISE

PROVE THE INTEGRAL FORMULAS ON P. 453-454 BY BOTH

a.) DIFFERENTIATION

b.) INTEGRATION (NOT POSSIBLE IN ALL CASES)

EX. PROVE #29 BY IBP.

$$\int x^n e^{ax}\,dx = \frac{1}{a}x^n e^{ax} - \frac{n}{a}\int x^{n-1}e^{ax}\,dx$$

EX. APPLY #29 3 TIMES TO:

$$\int x^3 e^{-x}\,dx = -e^{-x}\left(x^3+3x^2+6x+6\right) + C$$

EX. PROVE #10.

$$\int \frac{x}{ax+b}\,dx = \frac{x}{a} - \frac{b}{a^2}\ln|ax+b| + C$$

EX. USE #10 ON

$$\int \frac{2}{3x+4}\,dx = \frac{2x}{3} - \frac{8}{9}\ln|3x+4| + C$$

Ex use #17 & #16 on

$$\int \sqrt{9+4x^2}\,dx = x\sqrt{9+4x^2} + \frac{9}{2}\ln\left|2x+\sqrt{9+4x^2}\right| + C$$

Hw 7.6.1 (P. 457)

2-22 even

in the table; and there are many integrals that simply cannot be evaluated exactly and must be evaluated numerically. We will give a very brief list of indefinite integrals and explain how to use such tables.

I. Basic Functions

1. $\int x^n dx = \frac{1}{n+1}x^{n+1} + C, n \neq -1$
2. $\int \frac{1}{x} dx = \ln|x| + C$
3. $\int e^x dx = e^x + C$
4. $\int a^x dx = \frac{a^x}{\ln a} + C$ with $a > 0, a \neq 1$
5. $\int \ln x \, dx = x \ln x - x + C$
6. $\int \sin x \, dx = -\cos x + C$
7. $\int \cos x \, dx = \sin x + C$
8. $\int \tan x \, dx = -\ln|\cos x| + C$

II. Rational Functions

9. $\int \frac{1}{ax+b} dx = \frac{1}{a}\ln|ax+b| + C$
10. $\int \frac{x}{ax+b} dx = \frac{x}{a} - \frac{b}{a^2}\ln|ax+b| + C$
11. $\int \frac{x}{(ax+b)^2} dx = \frac{b}{a^2(ax+b)} + \frac{1}{a^2}\ln|ax+b| + C$
12. $\int \frac{x}{ax^2+bx+c} dx = \frac{1}{2a}\ln|ax^2+bx+c| - \frac{b}{2a}\int \frac{1}{ax^2+bx+c} dx$
13. $\int \frac{1}{a^2+x^2} dx = \frac{1}{a}\arctan\frac{x}{a} + C$
14. $\int \frac{1}{a^2-x^2} dx = \frac{1}{2a}\ln\left|\frac{x+a}{x-a}\right| + C$

III. Integrands Involving $\sqrt{a^2+x^2}$, $\sqrt{a^2-x^2}$, or $\sqrt{x^2-a^2}$

15. $\int \frac{1}{\sqrt{a^2-x^2}} dx = \arcsin\frac{x}{a} + C$
16. $\int \frac{1}{\sqrt{x^2 \pm a^2}} dx = \ln|x + \sqrt{x^2 \pm a^2}| + C$
17. $\int \sqrt{a^2 \pm x^2}\, dx = \frac{1}{2}\left(x\sqrt{a^2 \pm x^2} + a^2 \int \frac{1}{\sqrt{a^2 \pm x^2}} dx\right)$
18. $\int \sqrt{x^2 - a^2}\, dx = \frac{1}{2}\left(x\sqrt{x^2 - a^2} - a^2 \int \frac{1}{\sqrt{x^2 - a^2}} dx\right)$

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IV. Integrands Involving Trigonometric Functions

19. $\int \sin(ax)\,dx = -\frac{1}{a}\cos(ax) + C$

20. $\int \sin^2(ax)\,dx = \frac{1}{2}x - \frac{1}{4a}\sin(2ax) + C$

21. $\int \sin(ax)\sin(bx)\,dx = \frac{\sin(a-b)x}{2(a-b)} - \frac{\sin(a+b)x}{2(a+b)} + C, \text{ for } a^2 \neq b^2$

22. $\int \cos(ax)\,dx = \frac{1}{a}\sin(ax) + C$

23. $\int \cos^2(ax)\,dx = \frac{1}{2}x + \frac{1}{4a}\sin(2ax) + C$

24. $\int \cos(ax)\cos(bx)\,dx = \frac{\sin(a-b)x}{2(a-b)} + \frac{\sin(a+b)x}{2(a+b)} + C, \text{ for } a^2 \neq b^2$

25. $\int \sin(ax)\cos(ax)\,dx = \frac{1}{2a}\sin^2(ax) + C$

26. $\int \sin(ax)\cos(bx)\,dx = -\frac{\cos(a+b)x}{2(a+b)} - \frac{\cos(a-b)x}{2(a-b)} + C, \text{ for } a^2 \neq b^2$

V. Integrands Involving Exponential Functions

27. $\int e^{ax}\,dx = \frac{1}{a}e^{ax} + C$

28. $\int xe^{ax}\,dx = \frac{e^{ax}}{a^2}(ax-1) + C$

29. $\int x^n e^{ax}\,dx = \frac{1}{a}x^n e^{ax} - \frac{n}{a}\int x^{n-1}e^{ax}\,dx$

30. $\int e^{ax}\sin(bx)\,dx = \frac{e^{ax}}{a^2+b^2}(a\sin(bx) - b\cos(bx)) + C$

31. $\int e^{ax}\cos(bx)\,dx = \frac{e^{ax}}{a^2+b^2}(a\cos(bx) + b\sin(bx)) + C$

VI. Integrands Involving Logarithmic Functions

32. $\int \ln x\,dx = x\ln x - x + C$

33. $\int (\ln x)^2\,dx = x(\ln x)^2 - 2x\ln x + 2x + C$

34. $\int x^m \ln x\,dx = x^{m+1}\left[\frac{\ln x}{m+1} - \frac{1}{(m+1)^2}\right] + C, m \neq -1$

35. $\int \frac{\ln x}{x}\,dx = \frac{(\ln x)^2}{2} + C$

36. $\int \frac{1}{x\ln x}\,dx = \ln(\ln x) + C$

37. $\int \sin(\ln x)\,dx = \frac{x}{2}(\sin(\ln x) - \cos(\ln x)) + C$

38. $\int \cos(\ln x)\,dx = \frac{x}{2}(\sin(\ln x) + \cos(\ln x)) + C$