7.1.2 SUBSTITUTION: DEFINITE INTEGRALS

THERE ARE BASICALLY TWO WAYS TO TREAT DEFINITE INTEGRALS.

I. DETERMINE AN ANTIDERIVATIVE VIA SUBSTITUTION, THEN EVALUATE IT AT THE LIMITS OF INTEGRATION.

OR

II. TRANSLATE THE LIMITS OF INTEGRATION INTO THE NEW VARIABLE u, THEN EVALUATE THE ANTIDERIVATIVE (FUNCTION OF u) AT THESE LIMITS.

EX. $$\int_1^4 \sqrt{1+\tfrac{9}{4}x}\,dx = \ldots = \frac{8}{27}\left(1+\frac{9}{4}x\right)^{3/2}\Bigg|_1^4$$

$$= \frac{8}{27}\left(10^{3/2} - \left(\frac{13}{4}\right)^{3/2}\right)$$

TO TRANSLATE LIMITS OF INTEGRATION TO U-COORDINATES:

$$u = 1 + \frac{9}{4}x \Rightarrow \begin{cases} x = 4 \Rightarrow u = 10 \\ x = 1 \Rightarrow u = \dfrac{13}{4} \end{cases}$$

Thus

$$\int_1^4 \sqrt{1+\frac{9}{4}x}\,dx = \frac{4}{9}\int_{\frac{13}{4}}^{10} u^{1/2}\,du$$

$$= \frac{4}{9}\cdot\frac{2}{3}\cdot u^{3/2}\Big|_{13/4}^{10}$$

$$= \frac{8}{27}\left(10^{3/2} - \left(\frac{13}{4}\right)^{3/2}\right)$$

<u>Ex.</u>

$$\int_2^4 \frac{x^3-1}{x^4-4x}\,dx = \frac{1}{4}\int_8^{240}\frac{1}{u}\,du$$

$$= \frac{1}{4}\ln|u|\Big|_8^{240}$$

$u = x^4 - 4x$

$du = 4(x^3-1)\,dx$

$\frac{1}{4}du = (x^3-1)\,dx$

$$\begin{cases} x = 4 \Rightarrow u = 4^4 - 16 = 240 = 8\cdot 30 \\ x = 2 \Rightarrow u = 16 - 8 = 8 \end{cases}$$

$$= \frac{1}{4}\left(\ln 240 - \ln 8\right) = \frac{1}{4}\left(\ln(8\cdot 30) - \ln 8\right)$$

$$= \frac{1}{4}\left(\ln 8 + \ln 30 - \ln 8\right)$$

$$= \boxed{\frac{1}{4}\ln 30}$$

Ex. $\displaystyle\int_0^1 \frac{x^3}{\sqrt[4]{x^4+1}}\,dx = \int_0^1 x^3(x^4+1)^{-1/4}\,dx$

$u = x^4+1$
$du = 4x^3\,dx$
$\frac{1}{4}du = x^3\,dx$

$x=1 \leftrightarrow u=2$
$x=0 \leftrightarrow u=1$

$$= \int_1^2 u^{-1/4}\cdot\frac{1}{4}\,du$$

$$= \frac{1}{\cancel{4}}\cdot\frac{\cancel{4}}{3}u^{3/4}\Big|_1^2$$

$$= \boxed{\frac{1}{3}\left(2^{3/4}-1\right)}$$

Ex. $\displaystyle\int_5^9 \frac{x}{x-3}\,dx = \int_2^6 \frac{u+3}{u}\,du = \int_2^6 \left(1+\frac{3}{u}\right)du$

$u = x-3$
$du = dx$
$u+3 = x$

$x=9 \leftrightarrow u=6$
$x=5 \leftrightarrow u=2$

$$= \left(u+3\ln|u|\right)\Big|_2^6$$

$$= (6+3\ln 6)-(2+3\ln 2)$$

$$= 6+3(\cancel{\ln 2}+\ln 3)-2-3\cancel{\ln 2}$$

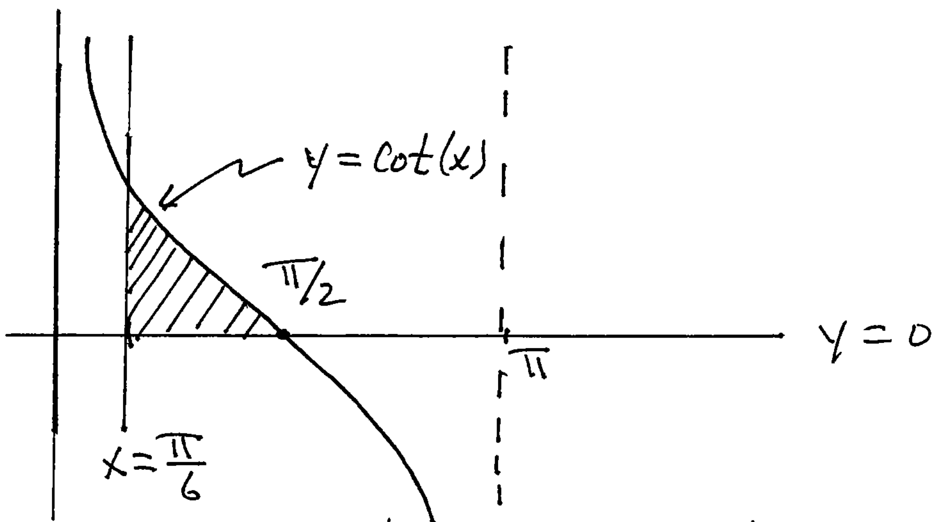
$$= \boxed{4+3\ln 3}$$

MORE EXAMPLES

$$\int_e^{e^2} \frac{dx}{x(\ln x)^2}$$

$$\int_1^9 \frac{1}{\sqrt{x}} e^{-\sqrt{x}}\, dx$$

EX FIND THE AREA BOUNDED BY
$y = \cot(x)$, $y = 0$, $x = \pi/6$



$$A = \int_{\pi/6}^{\pi/2} \cot(x)\,dx = \int_{\pi/6}^{\pi/2} \frac{\cos x}{\sin x}\,dx = \int_{1/2}^{1} \frac{1}{u}\,du$$

$u = \sin x$
$du = \cos x\,dx$
$x = \pi/2 \Leftrightarrow u = 1$
$x = \pi/6 \Leftrightarrow u = 1/2$

$$= \ln|u| \Big|_{1/2}^{1}$$

$$= \ln(1) - \ln\left(\tfrac{1}{2}\right)$$

$$= \boxed{\ln 2}$$

HW5 7.1.3 (P. 406)

2–36 even, 42–58 even