6.2.3 FTC (2^{nd} VERSION)

THE FTC (1^{st} VERSION) SAYS THAT

$$\int_a^x f(u)\,du$$

IS AN ANTIDERIVATIVE OF $f(x)$. LET $F(x)$ STAND FOR ANY OTHER ANTI-DERIVATIVE. THEN, AS WE SAW IN SEC. 5.8

$$F(x) = \int_a^x f(u)\,du + C$$

FOR SOME $C \in \mathbb{R}$. HENCE

$$F(b) - F(a) = \left[\int_a^b f(u)\,du + C\right] - \left[\int_a^a f(u)\,du + C\right]$$

$$= \int_a^b f(u)\,du + C - 0 + C$$

THEREFORE

$$\int_a^b f(u)\,du = F(b) - F(a).$$

WE REPLACE u BY x TO GET

Thm (FTC 2ND VERSION)

SUPPOSE $f(x)$ is CONTINUOUS ON $[a,b]$ AND $F(x)$ is ANY ANTIDERIVATIVE OF $f(x)$ (i.e. $F'(x) = f(x)$). THEN

$$\boxed{\int_a^b f(x)\,dx = F(b) - F(a)}$$

NOTATION: WE WRITE

$$F(x)\Big|_a^b = F(b) - F(a)$$

OR SOMETIMES

$$F(x)\Big|_{x=a}^{x=b} = F(b) - F(a).$$

Ex $\int_2^4 (3-2x)\,dx = 3x - x^2 \Big|_2^4$

$= (3\cdot 4 - 4^2) - (3\cdot 2 - 2^2) = \boxed{-6}$

Ex. $\int_0^1 (t^2 - \sqrt{t})\,dt$

Ex. $\int_1^8 s^{-2/3}\,ds$

Ex. $\int_0^2 (2x-1)(x+3)\,dx$

Ex. $\int_0^{\pi/8} \sec^2(2x)\,dx = \frac{1}{2}\tan(2x)\Big|_0^{\pi/8}$

$= \frac{1}{2}\tan\left(\frac{\pi}{4}\right) - \frac{1}{2}\tan(0)$

$= \frac{1}{2} - 0 = \boxed{\frac{1}{2}}$

REMEMBER:

DEFINITE INTEGRAL

$\int_a^b f(x)\,dx$ is A NUMBER

INDEFINITE INTEGRAL

$\int f(x)\,dx$ is A FAMILY OF FUNCTIONS

EX. $\int_0^1 (x^{3/5} + x^{5/3})\,dx$

VS.
EX. $\int (x^{3/5} + x^{5/3})\,dx$

HW 3 6.2.4 (P. 371)

2-10 even, 16-38 even, 44-64 even
100-120 even.