6.1.3 PROPERTIES OF DEFINITE INTEGRALS

1.) $\int_a^a f(x)\,dx$

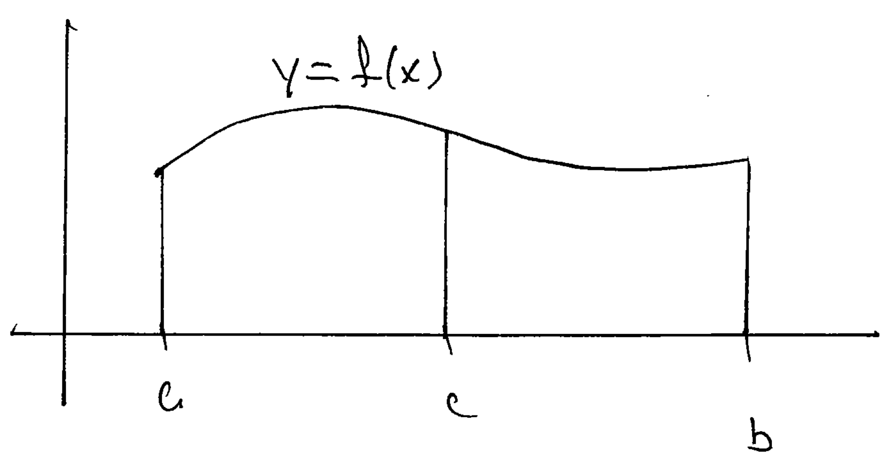
2.) $\int_b^a f(x)\,dx = -\int_a^b f(x)\,dx$

3.) $\int_a^b k\,f(x)\,dx = k\int_a^b f(x)\,dx$

4.) $\int_a^b [f(x)+g(x)]\,dx = \int_a^b f(x)\,dx + \int_a^b g(x)\,dx$

5.) IF $a, b, c \in I$, AND $f(x)$ INTEGRABLE ON I,

$$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$$



PROPERTIES 1 AND 2 ARE DEFINITIONS, WHILE 3, 4, AND 5 ARE PROVED USING THE DEFINITION OF THE DEFINITE INTEGRAL AS A LIMIT OF RIEMANN SUMS.

THE BOOK PROVES (4), SO WE SKETCH A PROOF OF (3).

CONSIDER ANY PARTITION $P = [x_0, \ldots, x_n]$ OF $[a,b]$ AND ANY $c_i \in [x_{i-1}, x_i]$ FOR $1 \le i \le n$. THE CORRESPONDING RIEMANN SUM FOR $k f(x)$ ON $[a,b]$ IS

$$\sum_{i=1}^{n} k f(c_i) \Delta x_i = k \sum_{i=1}^{n} f(c_i) \Delta x_i$$

TAKING LIMITS RIGHT AND LEFT AS $\|P\| \to 0$, WE GET

$$\int_a^b k f(x) dx = k \int_a^b f(x) dx$$

UPON PULLING k OUT OF THE LIMIT. ///

EX. DETERMINE $\int_a^b x^2\,dx$ FOR $0<a<b$.

RECALL, WE COMPUTED $\int_0^a x^2\,dx$ DIRECTLY AS A LIMIT OF RIEMANN SUMS TO BE

$$\int_0^a x^2\,dx = \frac{a^3}{3}$$

LIKEWISE

$$\int_0^b x^2\,dx = \frac{b^3}{3}$$

BY PROPERTY (5):

$$\int_0^b x^2\,dx = \int_0^a x^2\,dx + \int_a^b x^2\,dx$$

$$\therefore \int_a^b x^2\,dx = \int_0^b x^2\,dx - \int_0^a x^2\,dx$$

$$= \frac{b^3}{3} - \frac{a^3}{3} = \frac{b^3-a^3}{3}$$

NOTE: THIS IS PROBLEM 40.

MORE PROPERTIES

6.) IF $f(x) \geq 0$ ON $[a,b]$, THEN

$$\int_a^b f(x)\,dx \geq 0$$

7.) IF $f(x) \leq g(x)$ ON $[a,b]$, THEN

$$\int_a^b f(x)\,dx \leq \int_a^b g(x)\,dx$$

8.) IF $m \leq f(x) \leq M$ ON $[a,b]$ THEN

$$m(b-a) \leq \int_a^b f(x)\,dx \leq M(b-a)$$

(6) FOLLOWS FROM THE FACT THAT EVERY RECTANGLE WILL HAVE NON-NEGATIVE AREA, AND HENCE EVERY RIEMANN SUM WILL BE A NON-NEGATIVE NUMBER.

(7) FOLLOWS BY APPLYING (6) TO THE FUNCTION $g(x) - f(x) \geq 0$ TO GET

$$\int_a^b [g(x) - f(x)]\,dx \geq 0$$

THEN APPLYING PROPERTIES (3) & (4) TO GET

$$\int_a^b g(x)\,dx - \int_a^b f(x)\,dx \geq 0 .$$

(8) FOLLOWS FROM (7) AND THE FACT THAT

$$\int_a^b 1\,dx = b-a .$$

HW 2 6.1.4 (P. 355)

4, 6-36 even, 40-54 even, 62, 64
68abc, 72