CMPS 201
Spring 2010
Homework Assignment 4

1. (1 Point) Let $a \geq 1$, $b > 1$, $k \geq 0$, and consider the recurrence $T(n) = aT(n/b) + n^k$. Assume there exists $\varepsilon > 0$ such that $n^k = \Omega(n^{\log_b(a)+\varepsilon})$ for all sufficiently large *n*, so that if any case of the Master Theorem holds, it is case 3. Show that in this situation the regularity condition is necessarily satisfied, and hence by case 3, $T(n) = \Theta(n^k)$.

2. (1 Point) Define a function $T(n)$ for $n \geq 1$ by the recurrence

$$T(n) = \begin{cases} 1 & \text{if } n = 1 \\ (n-1) + \left(\dfrac{n-1}{n^2}\right) \cdot \displaystyle\sum_{k=1}^{n-1} T(k) & \text{if } n \geq 2 \end{cases}$$

   Use induction to show that $T(n) \leq 2n$ for all $n \geq 1$, and hence $T(n) = O(n)$.

3. (1 Points) Prove the correctness of Quicksort(*A*, *p*, *r*) by induction on the length $n = r - p + 1$ of the subarray $A[p \cdots r]$.