

CMPS 201
Spring 2010
Homework Assignment 3

1. (1 Point) Define $T(n)$ for $n \in \mathbf{Z}^+$ by the recurrence

$$T(n) = \begin{cases} 9 & \text{if } 1 \leq n < 15 \\ T(\lfloor n/2 \rfloor) + 6 & \text{if } n \geq 15 \end{cases}$$

Use the iteration method to find the exact solution to this recurrence, then find an asymptotic solution.

2. (1 Point) Define $T(n)$ for $n \in \mathbf{Z}^+$ by the recurrence

$$T(n) = \begin{cases} 4 & \text{if } n = 1, 2 \\ T(\lfloor n/3 \rfloor) + n & \text{if } n \geq 3 \end{cases}$$

Use the iteration method to find an asymptotic solution to this recurrence.

3. (3 Points) Consider the function $T(n)$ defined by the recurrence formula

$$T(n) = \begin{cases} 6 & 1 \leq n < 3 \\ 2T(\lfloor n/3 \rfloor) + n & n \geq 3 \end{cases}$$

- (1 Point) Use the iteration method to write a summation formula for $T(n)$.
 - (1 Point) Use the summation in (a) to show that $T(n) = O(n)$
 - (1 Point) Use the Master Theorem to show that $T(n) = \Theta(n)$
4. (5 Points) Use the Master Theorem to find tight asymptotic bounds on the following recurrences.
- (1 Point) $T(n) = 3T(2n/3) + n^3$
 - (1 Point) $T(n) = 2T(n/3) + \sqrt{n}$
 - (1 Point) $T(n) = 5T(n/4) + n^{\lg \sqrt{5}}$
 - (1 Point) $T(n) = 3T(2n/5) + n \lg(n)$
 - (1 Point) $S(n) = aS(n/4) + n^2$ (Note: your answer will depend on the parameter a .)

5. (2 Points) Consider the following algorithm:

WasteTime(n) (Precondition: n is a positive integer)

1. if $n \geq 2$
2. count $\leftarrow 0$
3. for $i \leftarrow 1$ to 7
4. WasteTime($\lfloor n/2 \rfloor$)
5. for $i \leftarrow 1$ to n^3
6. count \leftarrow count+1

Determine how much time WasteTime(n) wastes. For purposes of analysis, define line 6 to be a basic operation, and let $T(n)$ be the number of basic operations performed on the input $n \in \mathbb{Z}^+$, i.e. $T(n)$ is the total number of times line 6 is executed.

- a. (1 Point) Write a recurrence for $T(n)$.
- b. (1 Point) Find a tight asymptotic bound on $T(n)$ by any method.