CMPS 201
Spring 2010
Homework Assignment 1

1. (1 Point) Prove that if $f(n) = \Theta(g(n))$, then $f(n)^2 = \Theta(g(n)^2)$.

2. (1 Point) Let $f(n)$ and $g(n)$ be asymptotically positive functions. Prove that

$$f(n) + g(n) = \Theta(\max(f(n), g(n)))$$

3. (5 Points) List the following functions from lowest to highest asymptotic order. Indicate whether any two (or more) are of the same asymptotic order. Justify your answers.

$$2^{\lg(n)} \quad (\lg n)^{\lg n} \quad n^{1/\lg n} \quad \ln(\ln(n)) \quad 4^{\lg n} \quad \sqrt{2}^{\lg n} \quad n \quad \sqrt{2n} \quad 1$$
$$n^2 \quad n! \quad (3/2)^n \quad n^3 \quad (\lg n)^2 \quad 2^{2^n} \quad n2^n \quad n^{\lg(\lg n)} \quad \ln(n) \quad e^n$$
$$2^{\sqrt{2\lg(n)}} \quad 2^n \quad n\lg(n) \quad 2^{2^{n+1}} \quad \lg(n!) \quad \sqrt{\lg(n)} \quad 2^{(\lg n)^2} \quad (n+1)!$$

4. (2 Points) Let $f(n)$ and $g(n)$ be positive functions. Prove or disprove each of the following.

   a. (1 Point) If $f(n) = \Theta(g(n))$ then $2^{f(n)} = \Theta(2^{g(n)})$.

   b. (1 Point) If $f(n) = \Theta(g(n))$ then $\lg(f(n)) = \Theta(\lg(g(n)))$. Assume here that $\lim_{n\to\infty} g(n) = \infty$.

5. (2 Points) Determine the asymptotic order of each of the following expressions, i.e. for each expression, find a simple function $g(n)$ such that the expression is in the class $\Theta(g(n))$. Prove your answers.

   a. (1 Point) $\sum_{i=1}^{n} \log(i)$

   b. (1 Point) $\sum_{i=1}^{n} a^i$ where $a > 0$ is a constant. (Hint: consider the cases $0 < a < 1$, $a = 1$, and $a > 1$ separately.)

6. (1 Point) Use induction to prove that $\sum_{k=1}^{n} k^4 = \dfrac{n(n+1)(6n^3 + 9n^2 + n - 1)}{30}$ for all $n \geq 1$.