

ENAPS 12B

2-23-11

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• mid 1: wed mar 2
~~Tue mar 1~~

• review session: Tue mar 1
600 - 745 P
Phy Sci 114

• Assignment 4: Thur mar 3

Recursion

(2)

Ex. Binomial Coefficients

$C(n, k) = \#$ of k -subsets of an n -set. ($0 \leq k \leq n$)

$$S = \{1, 2, 3\} \quad n = 3$$

	<u>#</u>
0-subsets: \emptyset	1
1-subsets: $\{1\}, \{2\}, \{3\}$	3
2-subsets: $\{1, 2\}, \{1, 3\}, \{2, 3\}$	3
3-subsets: $S = \{1, 2, 3\}$	1

so

$$C(3, 0) = 1 = C(3, 3)$$

$$C(3, 1) = 3 = C(3, 2)$$

Recall Pascal's Δ :

<u>n</u>						
0	_____		1			
1	_____		1	1		
2	_____	1	2	1		
3	_____	1	3	3	1	
4	_____	1	4	6	4	1
⋮			⋮			

Thm: for $1 \leq k \leq n-1$

$$C(n, k) = C(n-1, k-1) + C(n-1, k)$$

Binomial Thm...

Ex. $(x+y)^4 = x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$

same for $(x+y)^n = \dots$

Ex

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// pre $n \geq 0, k \geq 0$

```
public static int C(int n, int k) {
```

```
    if (k > n)
```

```
        return 0;
```

```
    else if (k == 0 || k == n)
```

```
        return 1;
```

```
    else
```

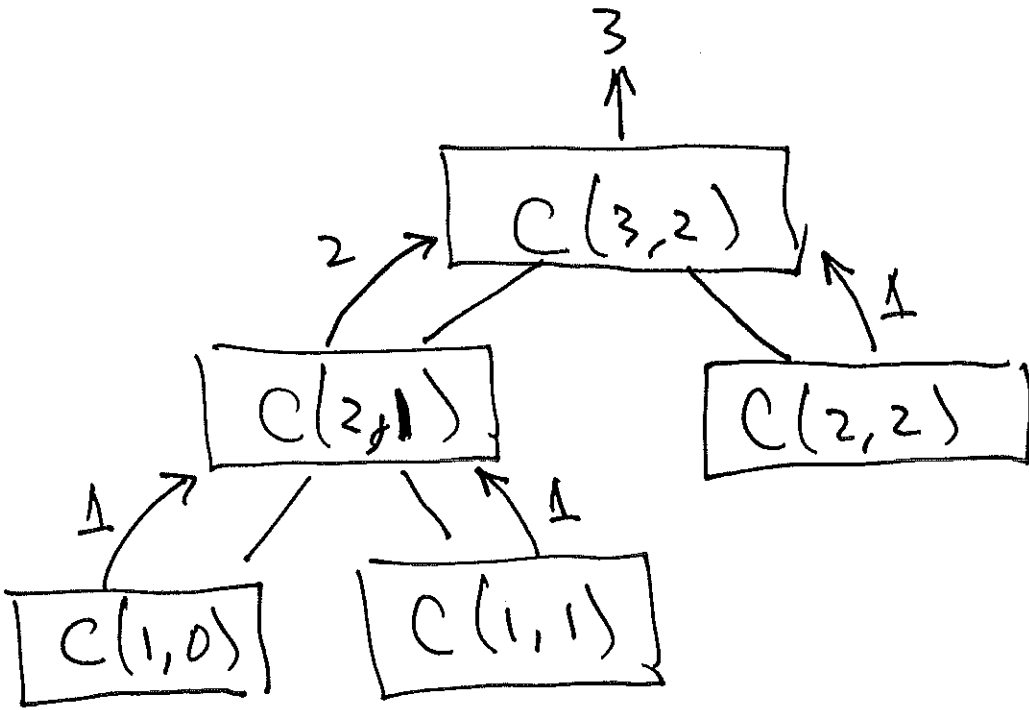
```
        return C(n-1, k-1) + C(n-1, k);
```

```
}
```

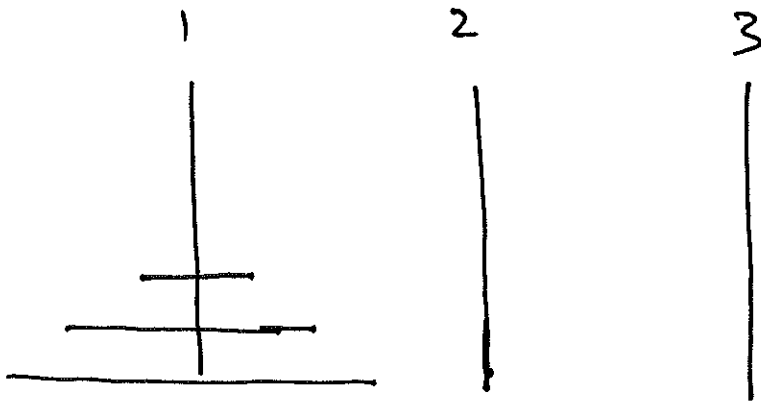
Exercise: envelope this in a class,

compute s, r, n .

Box trace of call: $C(3,2)$



EX. Towers of Hanoi



$n=3$: Solution: $1 \rightarrow 3, 1 \rightarrow 2, 3 \rightarrow 2$

$1 \rightarrow 3, 2 \rightarrow 1, 2 \rightarrow 3, 1 \rightarrow 3$

// Prints instructions for moving
 // n disks from rod i to rod j
 // pre: $n \geq 1, i, j \in \{1, 2, 3\}, i \neq j$

```

public static void Hanoi(int n, int i, int j) {
    int k = 6 - i - j;
    if (n == 1) {
        System.out.println(i + " -> " + j);
    } else {
        Hanoi(n-1, i, k);
        System.out.println(i + " -> " + j);
        Hanoi(n-1, k, j);
    }
}

```

Exercise: compile & run