RICHARD OTTE

A CRITIQUE OF SUPPES' THEORY OF PROBABILISTIC CAUSALITY*

An analysis of causality has been particularly troublesome, and thus mostly ignored, by those who believe the world is indeterministic. Patrick Suppes has attempted to give an account of causality that would hold in both deterministic and indeterministic worlds. To do this, Suppes uses probability relations to define causal relations. The main problems facing a probabilistic theory of causality are those of distinguishing between genuine and spurious causes as well as direct and indirect causes. Suppes presents several definitions of different types of causes in an attempt to capture the distinction between genuine and spurious causes and direct and indirect causes. It is my claim that Suppes' definitions fail to distinguish among genuine and spurious causes and direct and indirect causes. To support this claim I will give some counterexamples to Suppes' theory. I will then modify some of Suppes' definitions in a natural manner, and show that even with modification they are still prone to counterexamples.

The main thrust here is that Suppes' account of causation is intrinsically defective. I believe that there is no way to differentiate genuine from spurious causes or direct from indirect causes using only probability relations; thus no minor modifications of Suppes' definitions will be sufficient to resolve these difficulties. While presenting counterexamples to Suppes' definitions, I will also try to explain in principle why each particular example is a counterexample to Suppes' theory. After presenting these counterexamples, I will introduce the idea of an interactive fork and use it to argue that the basic intuition around which Suppes built his theory is faulty. In the last section of the paper I will discuss the more fundamental issue of whether all positive causes must raise the probabilities of their effects. Although this issue lies at the heart of most probabilistic accounts of causality, it has largely been ignored in the literature. I hope to show that we are not justified in believing that positive causes always raise the probability of their effects and that more discussion is needed on this important subject.

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SUPPES' THEORY

Suppes begins his discussion of causality by giving a definition of a *prima facie* cause:

Definition 1. The event $B_{t'}$ is a *prima facie* cause of the event A_t if and only if

(i) $t' < t$,
(ii) $P(B_{t'}) > 0$,
(iii) $P(A_t/B_{t'}) > P(A_t)$.[1]

The idea here is that it the probability of one event given another event is higher than the probability of the first event alone, then the two events are causally connected in some way. Being a *prima facie* cause is a prerequisite for being any kind of cause other than a negative cause. Throughout this discussion Suppes assumes that the direction of causation always follows the direction of time; he does not allow for reverse causation. However, even if two events are causally connected according to definition 1, we cannot infer that the earlier is the *cause* of the latter: they might both be effects of some common cause.

In order to account for this possibility, Suppes introduces the idea of a *spurious* cause:

Definition 2. An event $B_{t'}$ is a spurious cause of A_t (in sense one) if and only if $B_{t'}$ is a *prima facie* cause of A_t and there is a $t'' < t'$ and an event $C_{t''}$ such that

(i) $P(B_{t'}C_{t''}) > 0$,
(ii) $P(A_t/B_{t'}C_{t''}) = P(A_t/C_{t''})$,
(iii) $P(A_t/B_{t'}C_{t''}) \geq P(A_t/B_{t'})$. p. (23)

The intuitive idea here is that a spurious cause does not change the conditional probability of the event A_t given $C_{t''}$. The addition of $B_{t'}$ into the set of factors contributing to A_t has no real effect upon the occurrence of A_t; event $C_{t''}$ can account for event A_t at least as well as $B_{t'}$, can. The problem with this definition, however, is that it makes an event $B_{t'}$ spurious if an earlier event $C_{t''}$ satisfying the above requirements exists. Suppes himself finds this troublesome and develops a modified account of spuriousness.[2]

Suppes thinks that, if instead of demanding that an earlier event exists, we demand that a certain kind of earlier event exists, then we will have a more intuitive account of a spurious cause:

Definition 3. An event $B_{t'}$ is a spurious cause of A_t (in sense two) if and only if $B_{t'}$ is a *prima facie* cause of A_t and there is a $t'' < t'$ and a partition $\pi_{t''}$ such that for all elements $C_{t''}$ of $\pi_{t''}$

(i) $P(B_{t'}, C_{t''}) > 0$,

(ii) $P(A_t/B_{t'}C_{t''}) = P(A_t/C_{t''})$. (p. 25)[3]

This definition makes an event spurious$_2$ if the world can be partitioned in such a way that the above conditions are satisfied. Thus if we can observe a certain kind of event given by the partition, the observation of the later event $B_{t'}$ is uninformative, which makes it a spurious$_2$ cause. Suppes proves that if an event is a spurious$_2$ cause then it is a spurious$_1$ cause. The converse of this theorem, however, is not necessarily true: it is possible for an event to be a spurious$_1$ cause and not be a spurious$_2$ cause.

As an example of a spurious$_1$ cause, let us take the case of decreasing air pressure causing not only rain but also a falling barometer reading. The falling barometer reading is a *prima facie* cause of rain; given that the barometer reading is dropping, the probability that it will rain rises. Letting A denote rain, B denote a falling barometer reading, and C denote decreasing air pressure, the probability of rain given that the barometer reading and the air pressure are decreasing, $P(A/CB)$, is equal to the probability of rain given that the air pressure is decreasing, $P(A/C)$; thus the second condition of definition 2 is satisfied.[4] The third condition is likewise satisfied, since the probability of rain given decreasing air pressure and a falling barometer reading is at least as great as the probability of rain given a falling barometer reading, $P(A/BC) \geq P(A/B)$. Thus, by definition 2 a falling barometer reading is a spurious$_1$ cause of rain. The falling barometer reading is also a spurious$_2$ cause of rain. If we let π be our partition {decreasing air pressure, non-decreasing air pressure}, then

(i) $P(BC) > 0$,

(ii) $P(A/BC) = P(A/C)$,

(iii) $P(A/B\bar{C}) = P(A/\bar{C})$.

So the falling barometer reading is a spurious$_2$ cause of the rain.

Closely related to the notion of a spurious cause is the idea of an indirect cause. We will first define a *direct* cause:

Definition 4. An event $B_{t'}$ is a direct cause of A_t if and only if $B_{t'}$ is a *prima facie* cause of A_t and there is no t'' and no partition $\pi_{t''}$ such that for every $C_{t''}$ in $\pi_{t''}$

(i) $t' < t'' < t$,

(ii) $P(B_{t'}C_{t''}) > 0$,

(iii) $P(A_t/B_{t'}C_{t''}) = P(A_t/C_{t''})$. (p. 28)

We will then define an *indirect* cause to be a *prima facie* cause that is not direct. One immediately notices the similarity between definition 3 and definition 4. The main difference is that t'' falls between t and t' in definition 4. Although Suppes does not do so, this similarity suggests that a definition of direct cause could also be developed using the analysis of a spurious$_1$ cause.

Definition 5. An event $B_{t'}$ is a direct cause in sense one of A_t if and only if $B_{t'}$ is a *prima facie* cause of A_t and for every t'', $t' < t'' < t$, there is no $C_{t''}$ such that

(i) $P(B_{t'}C_{t''}) > 0$,

(ii) $P(A_t/B_{t'}C_{t''}) = P(A_t/C_{t''})$,

(iii) $P(A_t/B_{t'}C_{t''}) \geq P(A_t/B_{t'})$.

The conditions of definition 5 are similar to those of definition 2 with the difference that $t' < t'' < t$. We will call the definition of direct cause given by definition 4 direct cause *in sense two*.[5] Definitions 4 and 5 say that a cause is a direct cause if and only if there is no later event (or kind of event) that will account for A_t as well as $B_{t'}$ does. Whereas an event is direct$_2$ if a certain *kind* of event doesn't exist, an event is direct$_1$ if a certain *event* doesn't exist. This mirrors the difference between suprious$_1$ and spurious$_2$ causes. I mentioned earlier that Suppes proved that if a cause is a spurious$_2$ cause, then it is also a spurious$_1$ cause. A similar proof could be constructed to show that if a *prima facie* cause is a direct$_1$ cause, then it is also a direct$_2$ cause, and if it is an indirect$_2$ cause, then it is an indirect$_1$ cause.

Suppes then defines *supplementary* causes:

Definition 6. Events $B_{t'}$ and $C_{t''}$ are supplementary causes of A_t if and only if

(i) $B_{t'}$ is a *prima facie* cause of A_t,
(ii) $C_{t''}$ is a *prima facie* cause of A_t,
(iii) $P(B_{t'}C_{t''}) > 0$,
(iv) $P(A_t/B_{t'}C_{t''}) > \max(P(A_t/B_{t'}), P(A_t/C_{t''}))$. (p. 33)

Two causes are supplementary causes if the probability of an event occurring given both is higher than it would have been given either one alone. Thus, consuming drugs and consuming alcohol are supplementary causes of death, because the probability of dying given one has consumed drugs *and* alcohol is greater than either the probability of dying given one has consumed drugs *or* the probability of dying given one has consumed alcohol. It is worth noting that no spurious cause of A can be a supplementary cause of A. If, according to condition (ii) of definition 2 or 3, $P(A/BC) = P(A/C)$, then it is not the case that condition (iv) of definition 6 can be satisfied, so B and C will not be supplementary causes.

Sufficient causes are viewed as those limiting cases in which the conditional probability of an event reaches *one*:

Definition 7. An event $B_{t'}$ is a sufficient (or determining) cause of A_t if and only if $B_{t'}$ is a *prima facie* cause of A_t and $P(A_t/B_{t'}) = 1$. (p. 34)

Although some philosophers deny it, the sufficient cause relation is normally assumed to be transitive; if C is a sufficient cause of B, and B is a sufficient cause of A, then C is a sufficient cause of A. Suppes' analysis of a sufficient cause yields this result.[6]

Another important idea in causation is that of a necessary condition. Although Suppes does not discuss them, we can define a *necessary* cause as follows:

Definition 8. An event $B_{t'}$ is a necessary cause (or condition) of A_t if and only if $B_{t'}$ is a *prima facie* cause of A_t and $P(A_t/\bar{B}_{t'}) = 0$.

Event B is necessary for A if and only if the probability of A given the absence of B is equal to *zero*. We normally think that necessary causes or conditions are also transitive, and this analysis supports that idea.[7] In conjunction with the transitivity of sufficient causes, this entails that if we have a chain of necessary and sufficient causes, any member of that chain at t' is a necessary and sufficient cause of any member of that chain at t, for all $t > t'$.

SPURIOUS$_1$ AND DIRECT$_1$ CAUSES

I would like to assess the adequacy of these definitions with the use of several examples. Let us first consider the adequacy of the definition of a spurious$_1$ cause. It seems reasonable to believe the world is composed of both deterministic and probabilistic causes; presumably if there are indeterminate events they will be intermingled with determinate events and thus there will be causal chains consisting of both deterministic and probabilistic causes. For example 1, consider the causal chain $\longrightarrow D \longrightarrow C \longrightarrow B \dashrightarrow A$, where B is a probabilistic cause of A; C is a necessary and sufficient cause of B; D is a necessary and sufficient cause of C, etc. The first thing to notice is that B is a spurious$_1$ cause of A since the following conditions are satisfied:

(i) $P(BC) > 0$,
(ii) $P(A/BC) = P(A/C)$,
(iii) $P(A/BC) \geq P(A/B)$.[8]

This might lead us to ask if C is a genuine cause of A. But C is also a spurious$_1$ cause since

(i) $P(CD) > 0$,
(ii) $P(A/CD) = P(A/D)$,
(iii) $P(A/CD) \geq P(A/C)$,

are also true.[9] We could continue proving that each member of such a chain is a spurious$_1$ cause of A until we reached the first necessary and sufficient cause in that chain, if there is one. If there is a first necessary and sufficient cause in the chain, it will be the *only* genuine cause of A; if the chain has no first necessary and sufficient cause, then A has *no* genuine cause. Both these alternatives are paradoxical. Clearly A has *some* genuine causes, namely, all the members of that chain.

It is also important to notice that in example 1, B is a direct$_1$ cause of A; there is no event falling between A and B that renders B indirect$_1$. Furthermore, B is the only direct cause of A; C, D, and all of the other necessary and sufficient causes in the chain are indirect$_1$ since

(i) $P(BX) > 0$,
(ii) $P(A/BX) = P(A/B)$,
(iii) $P(A/BX) \geq P(A/X)$,

are satisfied, where X denotes any of the necessary and sufficient causes in the chain.[10] Thus B makes all of the other members of the chain indirect$_1$ causes of A, and all of the other members of the chain make B a spurious$_1$ cause of A. In this situation we have an earlier event making a later event, B, spurious$_1$, and the later event, B, making the earlier event indirect$_1$. Clearly this is an undesirable situation. We don't want to say the only direct$_1$ cause of A is spurious$_1$ and that the only genuine cause of A (if there is one) is indirect$_1$.[11] These problems arise because Suppes defines a cause to be spurious$_1$ if there exists an earlier event that can account for the probability of the effect just as well as any later event can. But in chains of necessary and sufficient causes, each member of a chain accounts for the probability of the last member just as well as any other member of that chain.

Example 2 concerns a situation in which it is possible for an effect to be caused as a result of *more than one* cause. Suppose we have a very brittle glass window, a gun and a slingshot, and that someone shoots the window with the slingshot while slightly later someone else shoots the window with the gun where the rock and bullet meet at the window and shatter it at just the same time. Let us adopt the following dictionary to symbolize this example:

W_t = the window being broken at t,
B_t = a bullet hitting the window at t,
R_t = a rock hitting the window at t,
G_t = a gun shot at the window at t,
S_t = a slingshot shot at the window at t.

The present example might then be diagrammed as follows:

$$\begin{tikzcd}[column sep=small, row sep=small]
 & & W_t & & \\
 & B_{t'} \arrow[ur] & & R_{t'} \arrow[ul] & \\
G_{t''} \arrow[ur] & & & & S_{t'''} \arrow[ul]
\end{tikzcd}$$

where $t''' < t'' < t' < t$. However, even though $B_{t'}$ and $R_{t'}$ are genuine causes of W_t, both $B_{t'}$ and $R_{t'}$ may be spurious$_1$ causes of W_t on Suppes' account. Suppose that the causes in this example are sufficient, i.e., that the following claims are true:

(i) $P(W_t/B_{t'}) = 1$,
(ii) $P(W_t/R_{t'}) = 1$,
(iii) $P(B_{t'}/B_{t''}) = 1$,
(iv) $P(R_{t'}/S_{t'''}) = 1$.

In this case both the bullet hitting the window ($B_{t'}$) and the rock hitting the window ($R_{t'}$) are spurious$_1$ causes.[12] But even more puzzling is the fact that shooting the gun at the window is a spurious$_1$ cause. The slingshot being shot at the window is the only genuine cause of the window breaking because:

(i) $P(G_{t''}S_{t'''}) > 0$,
(ii) $P(W_t/G_{t''}S_{t'''}) = P(W_t/S_{t'''}) = 1$,
(iii) $P(W_t/G_{t''}S_{t'''}) \geq P(W_t/G_{t''}) = 1$.

The only genuine reason shooting the slingshot is picked as the genuine cause instead of shooting the gun is that shooting the slingshot occurred earlier. But this seems to be ultimately arbitrary. I see no reason to believe that the earliest cause that gives a certain probability is therefore *the genuine cause.*

This becomes even more apparent if we change example 2 somewhat to get example 3. As before, let a slingshot be shot at the window, but in example 3 let the gun be fired at such a time that the bullet breaks the window and the rock follows immediately behind the bullet, having no impact on the glass. This example is the same as example 2 except that $B_{t'}$ is slightly before $R_{t'+\epsilon}$; thus, the rock has no effect on W_t. However, the same probability relations hold and shooting the gun is therefore a spurious$_1$ cause of the window breaking while the genuine cause is shooting the slingshot, even though it was the bullet that broke the window and the rock had no causal interaction with the window. This is an example in which the "spurious$_1$" cause is really the genuine cause, and the "genuine" cause (according to definition 2) is really a spurious cause. This result is the exact opposite of what we want; it arises because Suppes treats the earliest predictively informative cause as *the genuine cause,* which is not always appropriate. Suppes does not account for the time it may take a causal chain to occur and achieve its effect; in this case, for instance, trouble arises because the bullet travels much faster than does the rock. It appears that mere probability relations are not guaranteed to take this factor into account.[13]

In reply to these examples, Suppes could claim that, if we specify our events closely enough, there *is* a difference between a window broken by a rock and a window broken by a bullet. In that case the probability relations given above would not necessarily hold, and Suppes' definition of a spurious_1 cause might escape my criticism. I think that even if Suppes wanted to specify events that precisely, counterexamples could still be constructed. But even more importantly, I feel Suppes is committed to never specifying events that finely. Suppes wants his theory to account for our everyday use of the idea of causation, but in everyday language we do not ordinarily specify events very precisely. Thus if Suppes wishes to require that events be specified precisely in order to avoid these difficulties, he will no longer be able to claim that his theory captures our ordinary use of the concept of causation.

SPURIOUS_2 AND DIRECT_2 CAUSES

I would now like to discuss Suppes' analysis of spurious_2 causes. Let us continue the discussion of example 3 in which a gun is fired after a slingshot is shot, yet the bullet breaks the glass and the rock has no effect on the window. The shooting of the gun was spurious_1, but it is not spurious_2. In order for it to be spurious_2 the following would have to hold:

(i) $P(G_{t'}S_{t''}) > 0,\ P(G_{t'}\bar{S}_{t''}) > 0,$

(ii) $P(W_t/G_{t'}S_{t''}) = P(W_t/S_{t''}),$

(iii) $P(W_t/G_{t'}\bar{S}_{t''}) = P(W_t/\bar{S}_{t''}).$

But it should be clear that (iii) is not true. If the slingshot is not shot, the bullet shot by the gun will still break the window with a probability of 1, which is not equal to $P(W_t/\bar{S}_{t''})$. Thus the analysis of spurious_2 seems better off than the analysis of spurious_1, because the shooting of the gun in this example is not a spurious_2 cause. However, the shooting of the slingshot is still not considered a spurious_2 cause. But clearly the shooting of the slingshot *should* be considered a spurious cause in this case. Thus it appears as if the analysis of spurious_2, like that of spurious_1, looks for potential or possible causes without paying attention to actual causal chains. Furthermore, example 2 is not a counterexample to the definition of spurious$_2$, because if we take $\pi_{t'''}$ to be the natural partition $\{S_{t'''}\bar{S}_{t'''}\}$, then $P(W_t/G_{t''}\bar{S}_{t'''}) \neq P(W_t/\bar{S}_{t'''})$. If the slingshot is not shot, the shooting of the gun will cause the window to be broken.

The definition of spurious$_2$ also handles example 1. In order for B to be a spurious$_2$ cause of A, $P(BC)$ and $P(B\bar{C})$ must both be greater than zero. But if C is a necessary cause of B, $P(B\bar{C}) = 0$, and thus B is not a spurious$_2$ cause of A. The same reasoning shows that none of the members of the chain are spurious$_2$ causes. Suppes also proved a theorem showing that a partition $\{B, \bar{B}\}$ could not make C an indirect$_2$ cause, while the partition $\{C, \bar{C}\}$ made B a spurious$_2$ cause. (p. 29) Thus the definition of spurious$_2$ seems to have eluded the main thrust of example 1.

Although the criterion that, $P(BC) > 0$ for all members C of the partition π, saves the definition of spurious$_2$ from counterexamples that plague the definition of spurious$_1$, we shall see that this requirement brings along problems of its own. Consider example 4:

$A_{t''}$ = Joe's wife having syphilis at time t'',
$B_{t'}$ = Joe's having paresis at time t',
C_t = Joe's having syphilis at time t.

In this example, let Joe's having syphilis (C) be a common cause of his wife having syphilis (A) and Joe's having paresis (B). Let us further assume that syphilis is a necessary cause of paresis and that $t < t' < t''$. Joe's having paresis is then clearly a spurious cause of Joe's wife having syphilis, but unfortunately it is not a spurious$_2$ cause. In order for B to be a spurious$_2$ cause of A, we need a partition $\{C, \bar{C}\}$ for each member of which $P(BC) > 0$. But since C is a necessary cause of B, $P(B\bar{C}) = 0$. Thus B is not a spurious$_2$ cause of A, and Joe's having paresis is a genuine cause of his wife's having syphilis, which is not the case.

This situation will arise anytime a member of the partition is a necessary cause of the event we wish to be spurious$_2$. It may be clearer to look at a deterministic example. Suppose we have a very good barometer in which decreasing air pressure is a necessary and sufficient cause of a falling barometer reading. But then a falling barometer reading cannot be a spurious$_2$ cause of a storm, because the probability of a falling barometer reading *and* increasing air pressure is zero. But clearly this barometer is a spurious cause of the storm. Suppes' formulation of spurious_2 causes is too strong because it excludes effects of necessary causes from being spurious_2 causes *a priori*. This is a serious defect in his definition of spurious_2 causes.

Suppes does attempt to defend his requirement that, for all members of the partition π, $P(BC) > 0$. He says:

> An omniscient God might object to this aspect of the definition of spurious, but for limited human knowers it seems wholly defensible.... Only a God who knows everything would have a distribution that assigns only probability one or zero to any event, and only such a distribution could never satisfy the conditions. (p. 36)

One might expand Suppes' comments and object that Suppes' theory is a theory of *probabilistic* causality and therefore does not apply to cases in which probabilities reach limiting values of zero or one. Thus my counterexamples involving causes that are necessary or sufficient would be inappropriate.

In response to these objections, I would first note that Suppes clearly intended his theory to apply to cases in which *some* of the probabilities are one or zero. Suppes begins the exposition of his theory by remarking:

> It should be emphasized that the deterministic concept of cause prominent in classical physics simply occupies the place of a special case in the theory to be obtained here. Roughly speaking, we obtain the deterministic theory by letting all the probabilities in question be either 1 or 0. (p. 11)

Suppes also defined sufficient causes, which shows that he does not consider his theory inapplicable to cases where probabilities reach these extremal values. Thus his theory ought to be able to handle my counterexamples.

I also think that Suppes' comments indicate a tension, if not an inconsistency, in his theory. Suppes tends to vacillate between claiming his theory handles cases with probabilities of one and zero and claiming that there never are any such cases for limited human knowers. As a matter of fact, it is reasonable to demand that there be some continuity between an analysis of necessary and sufficient causation and an analysis of probabilistic causation. A probabilistic theory of causation should merge with traditional theories when probabilities reach zero or one. To satisfy this requirement we shouldn't have to deny necessary or sufficient causes exist, as Suppes appears to be doing.[14] Indeed, at other places in his monograph, Suppes suggests that we cannot deny that the necessary and sufficient causes of classical mechanics exist and that he intends his theory to account for them:

As I emphasize throughout this monograph, the theory of causality advanced here is not meant to be tailored to the latest physics. It is designed to provide a framework for the analysis of causality in a wide variety of theories and, hopefully, in a way that will usually fit the intuitions about causality that go with a given theory. (p. 34)

Suppes himself would reject the idea that an analysis of causality should determine which types of causes exist and don't exist. But he wanted his theory to be applicable to the idea of causation in all theories, including classical mechanics. All of this shows that Suppes' theory is faced with the problem of causes which are either necessary or sufficient for their effects. Thus, the definition of a spurious$_2$ cause, which requires that for all $C \in \pi$, $P(BC) > 0$, is seen to be unjustified.

We might find it profitable to attempt to modify the definition of spurious$_2$ so that it will encompass necessary causes, instead of completely rejecting the idea behind spurious$_2$ causes. If we ask why Suppes required that $P(BC) > 0$ for all members C of the partition π, the immediate answer is that, if $P(BC) = 0$ for some C, then $P(A/BC)$ is undefined, because dividing by zero is not allowed.[15] The intuition Suppes was trying to capture by his definition is that the occurrence of a spurious cause should have no real effect upon the probability of the effect occurring, once we have knowledge of the genuine cause. But if this is a correct intuition, it also seems as though the non-occurrence of a spurious cause should not affect the probability of the effect, given knowledge of the genuine cause. The following theorem supports this correlation:

THEOREM: If $P(BC) \neq 0$ and $P(\bar{B}C) \neq 0$, then $P(A/BC) = P(A/C)$ if and only if $P(A/\bar{B}C) = P(A/C)$.[16]

This theorem tells us that if $P(A/BC)$ and $P(A/\bar{B}C)$ are defined, B is irrelevant to $P(A/C)$ if and only if $\bar{B}$ is irrelevant to $P(A/C)$; thus, we might want to claim that B is a spurious cause of A if and only if $\bar{B}$ is a spurious cause of A. With this in mind let us consider a third definition of spurious causation.

DEFINITION: An event $B_{t'}$ is a spurious$_3$ cause of A_t if and only if $B_{t'}$ is a *prima facie* cause of A_t and there is a $t'' < t'$ and a partition $\pi_{t''}$

such that for all elements $C_{t''}$ of $\pi_{t''}$

(i) $P(B_{t'}C_{t''})>0 \rightarrow P(A_t/B_{t'}C_{t''})=P(A_t/C_{t''})$,

(ii) $P(\bar{B}_{t'}C_{t''})>0 \rightarrow P(A_t/\bar{B}_{t'}C_{t''})=P(A_t/C_{t''})$.

The definition of a spurious$_3$ cause allows B to be a spurious$_3$ cause of A, even if, for some member C of the partition π, C is a necessary cause of B(i.e. $P(B\bar{C})=0$).

After defining spurious$_3$ causes, we could now provide a definition of a direct$_3$ cause:

DEFINITION: An event $B_{t'}$ is a direct$_3$ cause of A_t if and only if $B_{t'}$ is a *prima facie* cause of A_t and there is no t'' and no partition $\pi_{t''}$ such that for every $C_{t''}$ in $\pi_{t''}$

(i) $t'<t''<t$,

(ii) $P(B_{t'}C_{t''})>0 \rightarrow P(A_t/C_{t''}B_{t'})=P(A_t/C_{t''})$,

(iii) $P(\bar{B}_{t'}C_{t''})>0 \rightarrow P(A_t/C_{t''}\bar{B}_{t'})=P(A_t/C_{t''})$.

We will then define a *prima facie* cause to be indirect$_3$ if it is not direct$_3$. Given these definitions the following relations hold:

(1) spurious$_2 \rightarrow$ spurious$_3 \rightarrow$ spurious$_1$

(2) spurious$_1 \not\rightarrow$ spurious$_3 \not\rightarrow$ spurious$_2$

(3) direct$_1 \rightarrow$ direct$_3 \rightarrow$ direct$_2$

(4) direct$_2 \not\rightarrow$ direct$_3 \not\rightarrow$ direct$_1$

(5) indirect$_2 \rightarrow$ indirect$_3 \rightarrow$ indirect$_1$[17]

This shows that spurious$_3$ is a stronger condition than spurious$_1$ but is a weaker condition than spurious$_2$.

The definition of spurious$_3$ handles examples 2 and 3 in the same manner spurious$_2$ does, so I won't repeat that discussion. But spurious$_3$ handles example 4 much better than spurious$_2$ did. Joe's having paresis is a spurious$_3$ cause of Joe's wife having syphilis, because the following is true:

$$P(A_{t''}/B_{t'}C_t)=P(A_{t''}/C_t),$$
$$P(A_{t''}/\bar{B}_{t'}\bar{C}_t)=P(A_{t''}/\bar{C}_t).$$

Thus the definition of spurious$_3$ handles example 4 the way it should and affords a more satisfactory definition than does that of spurious$_2$.

The requirement that for all members C of the partition π, $P(BC)>0$, enabled the definition of spurious$_2$ to escape the counterexample of example 1. The definition of spurious$_3$ relaxes that requirement somewhat and therefore cannot handle example 1 satisfactorily. The only genuine cause according to the definition of spurious$_3$ will be the *first* necessary and sufficient cause in the chain, if there is one; if the chain stretches infinitely into the past, A will have no genuine cause.[18] This is the same problem confronted by the definition of spurious$_1$.

The definition of indirect$_3$ also makes B the only direct cause of A, while all of the other members of the chain are indirect$_3$ causes. What happens is that the partition $\{B, \bar{B}\}$ makes C, D, etc. indirect$_3$, while the partition $\{C, \bar{C}\}$ makes B a spurious$_3$ cause.[19] Again, this is clearly undesirable. The definition of spurious$_3$ is no better off than the definition of spurious$_1$ in regard to example 1. The definition of spurious$_2$ was able to handle example 1 because it excluded many spurious causes from the class of spurious$_2$ causes. In the definition of spurious$_3$, we attempted to bring those spurious causes into the class of spurious$_3$ causes, but in doing so we also brought along many genuine causes.

The basic problem facing a definition of spurious causation which depends solely on probability relations among events seems to be that there is no way to distinguish between a causal chain and a fork when the probabilities involved are the same. Suppose we have the chain of necessary and sufficient causes $C \to B \to A$ and a necessary and sufficient common cause $B \nwarrow C \nearrow A$. All of the probability relations in these two cases will be identical, yet in one case B is a *genuine* cause of A and in another case B is a *spurious* cause of A. Thus there appears to be no way to distinguish genuine from spurious causes using only probability relations among the events, which is a very serious problem for a probabilistic theory of causality.

Another serious problem for Suppes' definitions of spurious causes is the existence of what Wesley Salmon has called *interactive* forks. Salmon characterizes an interactive fork ACB as one which satisfies the following conditions:

(i) $P(AB/C) > P(A/C)P(B/C)$,

(ii) $P(AB/\bar{C}) = P(A/\bar{C})P(B/\bar{C})$.

Thus the common cause C does not screen A and B off from one another. It is easily seen that (i) is equivalent to:

(iii) $P(A/BC) > P(A/C)$.

This condition guarantees in the interactive fork ACB that B is not a spurious cause of A in any of our three senses. The event B *is* predictively informative as to the occurrence of A; thus it is a genuine cause of A as well as often being a direct cause of A in one of the three senses, according to Suppes' theory. But it should be obvious that one prong of the fork is not really a genuine cause of the other prong and that Suppes' analysis errs at this point. Consider an example involving a disease and some side effect of the disease, such as a rash.[20] Let:

A_t = having a certain disease at t,
$B_{t'}$ = having a certain rash at t',
$C_{t''}$ = not being innoculated against the disease at t''.

In this case, it seems plausible that $P(A/BC) > P(A/C)$. Thus $B_{t'}$, i.e., having a certain rash at t', would be a genuine cause of A_t, i.e., having a certain disease at t, because the rash is predictively informative. Moreover, since both having the rash and not being innoculated *are* predictively informative, we can say that they are supplementary causes of having the disease. But clearly the rash is not a genuine cause of the disease at t even if it happens to be predictively informative.

Interactive forks are problematic for Suppes' theory of causation because the state of one of the prongs is the best way to predict the state of the other prong: the common cause of both prongs is *less informative* than knowledge of either of the prongs. But the intuition around which Suppes built his theory is that only genuine causes are predictively informative and that spurious causes are predictively uninformative. Interactive forks are not accounted for by Suppes' intuitions; thus, the more faithfully Suppes' theory reflects his intuition, the more serious will be the problem posed by interactive forks. Suppes has oversimplified by assuming that all genuine causes add predictive power and that all spurious causes are predictively uninformative.

Positive Statistical Relevance

I would now like to investigate whether a cause must raise the probability of its effect. Suppes has required that, in order to be a *prima facie* cause, an event must *raise* the probability of its effect. If a cause lowers the probability of its effect, it is a *negative* cause. To begin, consider an example due to Germund Hesslow, which seems to suggest that causes may lower the probabilities of their effects:

The basic idea in Suppes' theory is of course that a cause raises the probability of its effect, and it is difficult to see how the theory could be modified without upholding this thesis. It is possible however that examples could be found of causes that lower the probability of their effects. Such a situation could come about if a cause could lower the probability of other more efficient causes. It has been claimed, e.g., that contraceptive pills (C) can cause thrombosis (T), and that consequently there are cases where C_t caused $T_{t'}$. (The subscripts t and t' are Suppes' temporal indices.) But pregnancy can also cause thrombosis, and C lowers the probability of pregnancy. I do not know the values of $P(T)$ and $P(T/C)$ but it seems possible that $P(T/C) < P(T)$, and in a population which lacked other contraceptives this would appear a likely situation. Be that as it may, the point remains: *it is entirely possible that a cause should lower the probability of its effect.*[21]

I agree with Hesslow that it does appear possible for $P(T/C) < P(T)$. But that is only because taking contraceptives lowers the probability of pregnancy. Thus, it seems reasonable, for example, that

$$P(T/C\bar{P}) > P(T/\bar{P})$$

could be true, where P denotes being pregnant. Suppes' definition of a *prima facie* cause, however, can easily be relativized to background information, as he himself has observed. Thus, we might change the definition of *prima facie* cause to:

DEFINITION: $B_{t'}$ is a *prima facie* cause of A_t, with respect to information $C_{t'}$ if and only if

(i) $t' < t$,
(ii) $P(B_{t'}C_{t'}) > 0$,
(iii) $P(A_t/B_{t'}C_{t'}) > P(A_t/C_{t'})$. (p. 42)

In Hesslow's example we can let $\bar{P}$ be the background information. Then it follows that, with respect to that information, the taking of contraceptives (C) is a *prima facie* cause of thrombosis (T).

I think that this idea of Suppes becomes much more plausible in relation to the traditional notion of a causal field.[22] Some philosophers have claimed that all causal statements are made in reference to background information or assumptions called a *causal field.* It is really incorrect on this view to say that A caused B; what should be said instead is that in a certain causal field F, A causes B. An event A

may be a cause of B in a field F, yet not be a cause of B in field G. The natural counterpart of a causal field in probabilistic causality, moreover, is a reference class. An event may raise the probability of another event in one reference class and lower it in another. Thus, Suppes could reply to Hesslow by saying that, in certain reference classes or causal fields, the taking of contraceptives is a *prima facie* cause of thrombosis, while in other reference classes it is not.

With the notion of a causal field in mind, let us look at an example offered by Deborah Rosen, a former student of Suppes:

> . . . suppose a golfer makes a shot that hits a limb of a tree close to the green and is thereby deflected directly into the hole, for a spectacular birdie. Let the event to be explained, A_t, be the event of making a birdie, and let $B_{t'}$ be the event of hitting the limb earlier. If we know something about Mr. Jones' golf we can estimate the probability of his making a birdie on this particular hole. The probability will be low, but the seemingly disturbing thing is that if we estimate the conditional probability of his making a birdie, given that the ball hit the branch, that is, given that event $B_{t'}$ occurred, we would ordinarily estimate the probability as being still lower. Yet when we see the event happen, we recognize immediately that hitting the branch in exactly the way it did was essential to the ball's going into the cup. (p. 41)

In this example, we have a situation in which $P(A/B) < P(A)$. Thus B is not a *prima facie* cause of A, even though it was in the causal chain and seemed to be necessary for the event A. Rosen attempts to handle this by specifying the situation more closely in such a manner that B raises the probability of A occurring. What Rosen has done is specify a new narrower reference class in which B is a *prima facie* cause of A. We might characterize Rosen's method as that of locating a narrower reference class in which the event in question is a *prima facie* cause. Whether this is a legitimate method, however, is debatable.

Suppes claims that his account of probabilistic causation is supposed to account for the way in which we ordinarily use the idea of causation. But if this is true, the reference classes from which we take the probability of certain events do not have to be *homogeneous* reference classes. In the absence of objectively homogeneous reference classes, an event may be a negative *prima facie* cause with respect to a larger reference class and still be a *prima facie* cause with respect to a narrower reference class. Rosen has used this tack to escape the consequence that causes can lower the probability of their effects: when the event is placed in an objectively homogeneous reference class, it becomes a *prima facie* cause and raises the probability of its effect. But by using this method Rosen practically admits that in the ordinary use of the term "cause", a cause *can* lower the probability of its effect. We normally do not use homogeneous reference classes, of course, but rather rely upon reference classes made up of events we consider similar in some respect. Thus, it does not seem unusual that a cause may lower the probability of its effect, because we are using loosely constructed reference classes. Rosen may be correct in saying that, if we use objectively homogeneous reference classes, a cause always raises the probability of its effect. But, on the level of ordinary language, a cause does *not* always raise the probability of its effect. If Suppes wishes to hold onto the idea that a cause always raises the probability of its effect, therefore, he should restrict his theory to objectively homogeneous reference classes and forego his claim to capture the ordinary usage of language.

The question then remains whether, if we use only objectively homogeneous reference classes, causes always do raise the probability of their effects. Consider the following theorem:

Theorem; if B is a *prima facie* cause of A, then B is a *prima facie* negative cause of $\bar{A}$, i.e.:

$$P(A/B) > P(A) \longleftrightarrow P(\bar{A}/B) < P(\bar{A}).^{23}$$

This theorem is important in situations in which a *prima facie* cause occurs but an effect does not. Suppose that we have a true statistical law and that B is a *prima facie* cause of A. Since this is a statistical law, there will be times in which B occurs and A does not occur, or times in which B occurs and $\bar{A}$ occurs. But since B is a *prima facie* cause of A, it is a *prima facie* negative cause of $\bar{A}$. Thus we have B and $\bar{A}$ occurring, where B lowers the probability of $\bar{A}$. The important question for such cases is whether B is a cause of $\bar{A}$ or $\bar{A}$ has no cause and happened "by chance". I think either answer would be plausible. If we say $\bar{A}$ happens uncaused in this situation, we must be prepared to say that many events may happen uncaused. We might also say that sometimes B causes A and sometimes B causes $\bar{A}$. Ordinarily we speak as if an event B may cause A sometimes and other times cause $\bar{A}$. Consuming alcohol causes some people to become sleepy, but it causes others to become violent. I am not prepared to take a firm stand on either side of this issue, since each side has its appeal.

CONCLUDING REMARKS

In this paper I have attempted to present the basic features of Suppes' theory of probabilistic causality and to critically appraise the adequacy of that theory. I initially questioned his definitions of spurious causes. Since some causal chains have exactly the same probability relations as causal forks, it is impossible to distinguish between them using only probability relations. And it is then impossible to distinguish *genuine* causes from *spurious* causes using only probability relations. I have also questioned Suppes' intuition that spurious causes are predictively uninformative, once knowledge of a genuine cause is known. This is especially evident when interactive forks are considered. For interactive forks we found that either prong may be the best predictor of the state of the other, even though it is only a spurious cause of that prong. This shows that Suppes' basic intuition is at fault, since even if his theory were to capture his intuitions, it would still be unable to handle interactive forks. I then turned to the more fundamental issue of whether a cause always *raises* the probability of its effect, which Suppes presupposes throughout his entire theory. I claim that on the level of ordinary discourse we often do speak of causes which *lower* the probability of their effects. If we leave ordinary discourse to specify events more precisely, then it appears to be an open question as to whether a cause can actually lower the probability of its effect. This is a topic on which much more work needs to be done.[24]

The University of Arizona

NOTES

* The author wishes to thank Wesley Salmon for valuable comments on an earlier version of this paper.

[1] Suppes (1970), p. 12. Future references to this work will be denoted in the text by page number.

[2] Suppes neglects to explain precisely why he finds the analysis of a spurious cause in sense one troublesome. Perhaps the intuition he was trying to capture is that if C is a *genuine* cause of A and B a *spurious* cause of A, then the occurrence of B should be

irrelevant regardless of whether C occurs. The definition of a spurious cause in sense one does not reflect this intuition, which may have been the reason Suppes rejected it.

[3] From now on I will abbreviate "spurious in sense two" by "spurious$_2$" and "spurious in sense one" by "spurious$_1$."

[4] In this example and most of those to follow, I leave out temporal subscripts. They could easily be replaced if one so desired.

[5] From now on I will abbreviate "direct in sense one" by "direct$_1$" and "direct in sense two" by "direct$_2$."

[6] Theorem: If $P(A/B) > P(A)$, $P(B/C) > P(B)$, $P(A/B) = 1$, and $P(B/C) = 1$, then $P(A/C) = 1$.

Proof:

1. if $P(A/B) = 1$ and $P(BC) > 0$, then $P(A/BC) = 1$ — Suppes' theorem, p. 35.
2. $P(A/BC) = 1$ — 1. assumptions.
3. $P(A/C) = P(B/C)P(A/BC) + (\bar{B}/C)P(A/\bar{B}C)$ — Theorem on total probability.
4. $P(\bar{B}/C) = 0$ — from assumption that $P(B/C) = 1$
5. $P(A/C) = 1$ — 2, 3, 4.

[7] Theorem: If $P(A/B) > P(A)$, $P(B/C) > P(B)$, $P(A/\bar{B}) = 0$, and $P(B/\bar{C}) = 0$, then $P(A/\bar{C}) = 0$.

Proof:

1. if $P(A/\bar{B}) = 0$ and $P(\bar{B}\bar{C}) > 0$, then $P(A/\bar{B}\bar{C}) = 0$. — Theorem similar to Suppes' theorem, above.
2. $P(A/\bar{B}\bar{C}) = 0$ — 1. assumptions
3. $P(A/\bar{C}) = P(B/\bar{C})P(A/B\bar{C}) + P(\bar{B}/\bar{C})P(A/\bar{B}\bar{C})$ — Theorem on total probability.
4. $P(A/\bar{C}) = 0$ — 2, 3, assumptions

[8] (i) Condition (i) is satisfied because $P(B/C) = 1$.

(ii) Condition (ii) is satisfied because $P(B/C) = 1$:

1. $P(B/C) = 1$ — assumption
2. $P(B/AC) = 1$ — 1, Suppes' Theorem
3. $P(BCA) = P(CA)$ — 2, definition
4. $P(BC) = P(C)$ — 1, definition
5. $\dfrac{P(ABC)}{P(BC)} = \dfrac{P(AC)}{P(C)}$ — 3, 4
6. $P(A/BC) = P(A/C)$ — 5

(iii) Condition (iii) is satisfied because $P(B/\bar{C}) = 0$:

1. $P(B/\bar{C}) = 0$ — assumption
2. $P(B\bar{C}) = 0$ — 1, definition
3. $P(AB\bar{C}) = 0$ — 2
4. $P(AB) = P(ABC) + P(AB\bar{C})$ — Theorem
5. $P(AB) = P(ABC)$ — 3, 4
6. $P(BC) = P(B) - P(B\bar{C})$ — Theorem
7. $P(BC) = P(B)$ — 2, 6
8. $P(A/BC) = P(A/B)$ — 5, 7, definition

[9] The proof of this follows from the transitivity of sufficient and necessary causes. Since $P(B/C) = 1$ and $P(C/D) = 1$, we know that $P(B/D) = 1$. Similarly we know that $P(B/\bar{D}) = 0$. Thus we know that C is a spurious$_1$ cause of A from the above proofs, *mutatis mutandis*.

[10] The proof of this is similar to those given in notes 8 and 9 above.
[11] Suppes believed that spurious causes of A should not make genuine causes of A *indirect causes*, and he proved it could not happen for spurious$_2$ and indirect$_2$ causes.
[12] $B_{t'}$ is a spurious$_1$ cause of W_t because $G_{t''}$ occurs earlier than $B_{t'}$ and

(i) $P(B_{t'}G_{t''}) > 0$,

(ii) $P(W_t/B_{t'}G_{t''}) = P(W_t/G_{t''}) = 1$,

(iii) $P(W_t/B_{t'}G_{t''}) \geq P(W_t/B_{t'}) = 1$,

are true. $R_{t'}$ is a spurious$_1$ cause of W_t because $S_{t'''}$ occurs earlier than $R_{t'}$ and

(i) $P(R_{t'}S_{t'''}) > 0$,

(ii) $P(W_t/R_{t'}S_{t'''}) = P(W_t/S_{t'''}) = 1$,

(iii) $P(W_t/R_{t'}S_{t'''}) \geq P(W_t/R_{t'}) = 1$,

are true.
[13] This example also presents a problem for the definition of *supplementary causes* (definition 6). Suppose all of the causes are not sufficient but instead are probabilistic. In this case, shooting the gun and shooting the slingshot are supplementary causes of the window breaking. But if the bullet breaks the window, it is false to say that shooting the slingshot is a genuine supplementary cause. This supports the claim that probability relations alone cannot pick out actual causal chains.
[14] It appears as though Reichenbach faced a similar tension when he developed his theory of probabilistic causality. See Reichenbach (1956).
[15] I am assuming here the traditional definition of conditional probabilities, i.e.

$$P(A/B) = \frac{P(AB)}{P(B)}.$$

[16] Proof: Suppose $P(BC) \neq 0$ and $P(\bar{B}C) \neq 0$

1.	$P(A/BC) = P(A/C)$	assumption
2.	$\dfrac{P(ABC)}{P(BC)} = \dfrac{P(AC)}{P(C)}$	1
3.	$P(C)P(ABC) = P(AC)P(BC)$	2
4.	$P(C)P(ABC) - P(ABC)P(BC) = P(AC)P(BC) - P(ABC)P(BC)$	3
5.	$P(ABC)(P(C) - P(BC)) = P(BC)(P(AC) - P(ABC))$	4
6.	$P(ABC)P(\bar{B}C) = P(BC)P(A\bar{B}C)$	5
7.	$\dfrac{P(ABC)}{P(BC)} = \dfrac{P(A\bar{B}C)}{P(\bar{B}C)}$	6
8.	$P(A/BC) = P(A/\bar{B}C)$	7

Thus if $P(BC) \neq 0$ and $P(\bar{B}C) \neq 0$, then $P(A/BC) = P(A/C)$ if and only if $P(A/\bar{B}C) = P(A/C)$.
[17] Proofs:

1*a*. Spurious$_2 \rightarrow$ spurious$_3$. Assume B is a spurious$_2$ cause of A because of π. Then for all $C \in \pi$,

(i) $P(BC) > 0$,

(ii) $P(A/BC) = P(A/C)$.

Thus condition (i) of the definition of spurious$_3$ is satisfied. If $P(\bar{B}C)=0$ for some C, then condition (ii) is satisfied for that C. If $P(\bar{B}C)>0$ for some C, then the consequent of condition (ii) is also satisfied by the above theorem. Thus spurious$_2 \rightarrow$ spurious$_3$.

1*b*. Spurious$_3 \rightarrow$ spurious$_1$. Suppose B is a spurious$_3$ cause of A because of $\{C, \bar{C}\}$. Then we have 4 possible cases to look at.

Case 1. $P(BC)>0$ and $P(B\bar{C})>0$. If this is true, then B is also a spurious$_2$ cause of A. But Suppes has proven that all spurious$_2$ causes are spurious$_1$ causes. (p. 25) Thus B is a spurious$_1$ cause.

Case 2. $P(BC)>0$ and $P(B\bar{C})=0$. Since B is a spurious$_3$ cause, $P(A/BC)=P(A/C)$, so condition (ii) of definition 2 is satisfied. $P(A/BC)=\frac{P(ABC)}{P(BC)}$. Also, $P(BC)=P(B)-P(B\bar{C})=P(B)$, since $P(B\bar{C})=0$. $P(ABC)=P(AB)-P(AB\bar{C})=P(AB)$, since $P(B\bar{C})=0$. So $\frac{P(ABC)}{P(BC)}=\frac{P(AB)}{P(B)}$. So $P(A/BC)=P(A/B)$, and condition (iii) is satisfied.

Case 3. $P(BC)=0$ and $P(B\bar{C})>0$. The proof of this is the same as for case 2 except we show that $P(A/B\bar{C})=P(A/\bar{C})$ and $P(A/B\bar{C})=P(A/B)$.

Case 4. $P(BC)=0$ and $P(B\bar{C})=0$. But $P(B)=P(BC)+P(B\bar{C})=0$, which is false, since B is a *prima facie* cause of A. So this is not a legitimate case. Thus spurious$_3 \rightarrow$ spurious$_1$.

2*a*. Spurious$_1 \not\rightarrow$ spurious$_3$. Let A = a disease; C = an in injection of bacteria known to cause A; and B = being exposed to people who have this disease. Suppose $P(A/C)=1$, $P(A/\bar{C})$ is very low, and $P(A/B)=\frac{1}{2}$. Then $P(A/BC)=P(A/C)\geq P(A/B)$, so B is a spurious$_1$ cause of A. But B is not a spurious$_3$ cause, since $P(A/B\bar{C})\neq P(A/\bar{C})$. So being a spurious$_1$ cause does not imply being a spurious$_3$ cause.

2*b*. Spurious$_3 \not\rightarrow$ spurious$_2$. In example 4, Joe's having paresis was not a spurious$_2$ cause of his wife's having syphilis, but it is a spurious$_3$ cause. Thus some spurious$_3$ causes are not spurious$_2$ causes.

Since the definitions of direct and indirect causes mirror the definitions of spurious causes, the rest of the table can be proved *mutatis mutandis*.

[18] Theorem: If $P(B/\bar{C})=0$, then $P(\bar{B}/\bar{C})=1$. So consider any member of a necessary and sufficient causal chain, say G. If I is a member of the chain before G, the following holds:

(i) $P(GI)>0$, $P(\bar{G}I)=0$, $P(G\bar{I})=0$, $P(\bar{G}\bar{I})>0$,

(ii) $P(A/GI)=P(A/I)$,

(iii) $P(A/\bar{G}\bar{I})=P(A/\bar{I})$.

That these hold is essentially a result of the above theorem, the transitivity of necessary and sufficient causes, and the results of note 9, part (ii). If there is no member of the chain before G, G will be the first necessary and sufficient cause in the chain and the only genuine cause of A. If there is no first cause, then there is no genuine cause of A.

[19] The partition $\{B, \bar{B}\}$ makes a previous member of the chain, say X, indirect$_3$, since the following hold:

(i) $P(A/BX)=P(A/B)$,

(ii) $P(A/\bar{B}\bar{X}) = P(A/\bar{B})$.

The proof of this is essentially the same as that given in note 16, *mutatis mutandis*.

[20] I am indebted to Paul Humphreys for first thinking of this example.

[21] Hesslow (1976), pp. 290–292.

[22] See Mackie (1974) for a discussion of the idea of a causal field.

[23] Proof:

1. $P(A/B) > P(A)$	Assumption
2. $P(AB) = P(B) - P(B\bar{A})$	theorem
3. $\frac{P(AB)}{P(B)} > P(A)$	1
4. $\frac{P(B) - P(B\bar{A})}{P(B)} > P(A)$	2, 3
5. $1 - \frac{P(B\bar{A})}{P(B)} > P(A)$	4
6. $1 - P(\bar{A}/B) > P(A)$	5
7. $1 - P(\bar{A}/B) > 1 - P(\bar{A})$	6
8. $P(\bar{A}) > P(\bar{A}/B)$	7

[24] For further discussion of this problem see Salmon (1980). For a counterfactual analysis of probabilistic causality, see Fetzer and Nute (1979) and (1980).

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