

Problem Set 4

Due: Tuesday, November 20, 2012

1. In an economy, a representative household has the utility function,

$$U_0 = \sum_{t=0}^{\infty} \beta^t u(c_t),$$

and the budget identity,

$$a_{t+1} = (1 + r_t) a_t + w_t - T_t - c_t,$$

where T_t equals lump-sum taxes paid at time t and w_t is labor income. Firms produce output according to the production function, $y_t = f(k_t)$, and capital depreciates at the rate δ . All variables are per capita. There is a government that makes a sequence of expenditures, g_t , that do not contribute to household utility or firm productivity.

a) Suppose that the government balances its budget each period (that is, $g_t = T_t$ for all $t \geq 0$). Complete the optimization problem for the household and find the conditions that determine its savings and consumption. Give the conditions that determine output supply and factor demands for firms. Add the equilibrium conditions so that you have a complete specification of the decentralized equilibrium.

b) Use your equations to show how a permanent increase in g_t from the constant, g , to the new level $\hat{g} > g$, affects the equilibrium. Assume the economy is initially in steady state.

c) Do the same for a temporary increase in g_0 from g to \hat{g} . Give a short interpretation of how government spending affects the decentralized equilibrium.

2. Use the economy from question 1, but now let the government can issue debt. The government's budget identity is

$$b_{t+1} = (1 + r_t) b_t + g_t - T_t.$$

a) Specify the decentralized equilibrium with government debt. You will need to redo the equilibrium conditions. The rest of your problem should not change. How does household optimization impose a solvency constraint on the government? If you impose the solvency condition on the household, does the household optimum imply the government must be solvent? Show.

b) Solve the government budget identity forward and impose the solvency condition for the government (this should be an inequality). To make the algebra simpler to write, assume the production function is $f(k_t) = (r + \delta) k_t$. This is the government's budget constraint. Taking the sequence $\{g_t\}$ as fixed with initial debt, b_0 , predetermined, what requirement does government solvency impose on tax revenue?

c) Solve the household budget identity forward. Show that only the present value of taxes affects household consumption at any time. Recognize that this means that only the initial debt and present value of

government spending affect household consumption.

d) Use the budget identities and equilibrium conditions to show that only g_t affects capital accumulation and consumption in this economy. You should recognize that the timing of lump-sum taxes does not affect the equilibrium allocation for the economy. Make sure you see this.

3. A simple way to introduce productive public spending is to allow public goods in production in an endogenous growth model. Let the production function for firms be

$$y_t = Ak_t^\alpha g_t^{1-\alpha}$$

and let the rate of depreciation be zero. Assume g_t is government consumption (that is, not investment).

Use continuous time and let household utility be

$$U_t = \int_t^\infty e^{-\theta(s-t)} \frac{c_t^{1-\sigma} - 1}{1-\sigma} ds$$

to make the analytics easy.

a) For a given fiscal policy with lump-sum taxation, find the relationships that determine a decentralized equilibrium. Make sure you include profit distributions to households and notice that labor is not a factor in this economy.

b) Solve out for equilibrium for the case that g_t is some constant proportion of the capital stock ($\frac{g_t}{k_t} > 0$ is constant). What is the marginal product of capital and growth rate in the balanced growth path?

c) Find the command optimum by including g_t in the control set. How does g_t depend on k_t ? What is the marginal product of capital and growth rate in the optimum? (Is the optimum a balanced growth path for all t ?)

d) What are equilibrium profits (that is, their interpretation)? How are these related to g_t in the optimum? Confirm that a balanced budget lump-sum tax is identical to a one hundred percent tax on profit.

4. This problem uses the same economy as problem 3 but rules out lump-sum taxation.

a) Suppose the government can only tax output at a proportionate rate. Tax revenues at time t equal $\tau_t y_t$ where τ_t is the tax rate. Repeat setting up the decentralized equilibrium in this case. The tax rate and government expenditures are given, but they must satisfy the government's budget constraint.

b) Find the conditions for the equilibrium for this economy for a balanced budget policy. Solve these for the balanced growth path assuming the share of g_t in output is constant.

c) Compute household welfare in the balanced growth path. Then maximize it with respect to the share of government spending in output for the balanced budget policy.

d) The policy you found in part c is the optimal fiscal policy under proportionate output taxation. Compare this constrained optimal growth rate to the growth rate you found in problem 3.