

Problem Set 3

Due: Thursday, November 1, 2012

1. Consider an economy with external economies of scale in production in which capital is the only factor. There are lots of firms, and the productivity of capital for an individual firm rises with the economy-wide capital stock. The production function for firm j is given by

$$y_j = Ak_j^\alpha \kappa^{1-\alpha},$$

where k_j is the capital stock of firm j , κ is the average capital stock per firm and $0 < \alpha < 1$. A representative household seeks to maximize

$$U_t = \int_t^\infty e^{-\theta(s-t)} \log c_s ds$$

given its flow budget identity,

$$\dot{a}_t = r_t a_t + \pi_t - c_t,$$

where a_t equals financial wealth per capita, π_t equals per capita profit income and c_t is per capita consumption. Assume that the population is constant and that capital is the only financial asset. y_j , k_j and κ are all per capita variables for convenience.

a) Taking r and π as given, set up the optimal savings problem for the household and find the necessary conditions for the household optimum.

b) Assume that each firm is small enough that it takes κ and r as given when it makes its optimal choice of k_j . What is k_j in equilibrium for the firm? Remember that the interest rate r is the opportunity cost of capital for the firm. Will k_j be the same for all firms? Using your answer, what are the equilibrium interest rate, per capita profit income and per capita output given total per capita capital, k_t ?

c) Solve for the market equilibrium for this economy. Is it a balanced growth path? At what rate does the economy grow? If any parameter restrictions are required for your solution, point these out.

d) Now, set up the problem for finding the command optimum for this economy. Solve for the optimal growth rate and for consumption as a function of k_t in the optimum. Compare these to your answers for part (c) (why is the equilibrium you found in part (c) not optimal?).

2. Consider household consumption and saving with durable and nondurable goods in continuous time. The household seeks to maximize the utility function,

$$U_t = \int_t^\infty u(c_s, D_s) e^{-\theta(s-t)} ds,$$

where c denotes consumption of nondurables and D denotes the stock of durables of which the household

derives utility from a proportional flow of services. The function in the integrand is

$$u(c_s, D_s) = \gamma \log c_s + (1 - \gamma) \log D_s.$$

The household receives labor income and accumulates savings at a constant rate of interest, r . Durables are accumulated according to

$$\dot{D} = d_t - \delta D_t,$$

where d denotes the flow of durable goods purchases. The relative price of durables at time t is p_t .

- a) Set up the optimization problem for the household and derive the necessary conditions for an optimum.
- b) Use your equations to simplify the equilibrium dynamics for consumption. You do not need to assume that $\theta = r$.
- c) Write out equilibrium condition for the ratio of nondurables goods consumption in terms of durables consumption. Use this to identify the shadow price of durables goods in terms of nondurables from your model assuming that p is not constant over time. Interpret this price.
- d) In the steady state (now, let $r = \theta$), show how a sudden increase in p affects consumption shares on durables and nondurables. Explain how consumption shares respond if p rises suddenly but then falls back to its original value?

3. Consider household consumption and saving in discrete time where the household seeks to maximize

$$U_t = \sum_{s=t}^{\infty} \beta^{s-t} u(c_s)$$

subject to the budget identity,

$$a_{s+1} - a_s = r a_s + w_s - c_s.$$

- a) Complete the optimization problem and solve for consumption c_t given that $\beta = (1 + r)^{-1}$.
- b) Suppose that $w_t = w + \varepsilon$ and $w_s = w$ for all $s > t$ where $\varepsilon > 0$. Show how c_t depends on ε . Suppose instead that $w_s = w + \varepsilon$ for all $s \geq t$. Show how c_t depends on ε . Interpret your results in terms of temporary and permanent unanticipated increase in labor income.
- c) Suppose that labor income follows the process

$$w_{s+1} - w = \rho(w_s - w)$$

and that $w_t = w + \varepsilon$. Show how c_t depends on ρ . Interpret your answer in terms of the persistence of an unanticipated rise in labor income.

- d) Suppose that the household suddenly learns that labor income will rise in the future. Specifically, at date t , the household learns that earnings will rise from w to $w + \varepsilon$ at date $t' > t$. Show how this affects consumption at date t and at date t' .