

Problem Set 3
Due: Thursday, May 2

1. Use the combined Solow-Romer model explained in the appendix to Chapter 6 to answer the following.

a) How does an increase in the rate of depreciation from $\bar{d}$ to $\bar{d}'$ change the long-run (balanced) growth rate?

b) Explain the transition dynamics after $\bar{d}$ rises if the economy begins in balanced growth.

c) How does an increase in the productivity of R and D from $\bar{z}$ to $\bar{z}'$ change the growth rate of per capita output, y^* ?

d) Explain the transition dynamics for the increase in $\bar{z}$.

2. Suppose that human capital is what makes labor productive. In this case, the production function for the economy is

$$Y_t = \bar{A} K_t^{1/3} H_t^{2/3}.$$

Human capital is produced through education, and investment is divided between investment in capital and in human capital (education). The capital accumulation equation is

$$\Delta K_{t+1} = \bar{s}_K Y_t - \bar{d} K_t,$$

and the education equation is

$$\Delta H_{t+1} = \bar{s}_H Y_t - \bar{d} H_t.$$

In this economy, the overall investment rate is $\bar{s}_K + \bar{s}_H$. $\bar{A}$ is constant.

In the long run, the economy follows a balanced growth path. The following steps will help you find it.

a) Start by setting the growth rates of capital and human capital equal using the capital accumulation and the education equations. Use this equality to derive an equation relating the output to capital ratio (Y/K) to the output to human capital ratio (Y/H).

b) Simplify your solution to part a) by eliminating Y and rearranging to get that

$$H_t = \frac{\bar{s}_H}{\bar{s}_K} K_t$$

along the balanced growth path. Substitute this relationship into the production function to eliminate H . You should get

$$Y_t = \bar{A} K_t \left(\frac{\bar{s}_H}{\bar{s}_K} \right)^{2/3}.$$

c) Does this solution show that the growth rate of GDP equals the growth rate of capital? To find the growth rate, substitute this solution for GDP into the capital accumulation equation and solve for the growth rate of K. Show that the balanced growth rate is

$$g_Y = \bar{s}_K \bar{A} \left(\frac{\bar{s}_H}{\bar{s}_K} \right)^{2/3} - \bar{d} = \bar{A} \bar{s}_K^{1/3} \bar{s}_H^{2/3} - \bar{d}.$$

d) In this model, how does an increase in the rate of investment in education affect long-run growth of GDP? How does an increase in the rate of investment in capital affect the long-run growth rate of GDP?

e) Draw a graph of the logarithm of GDP to show an increase in the investment rate in education affects GDP.

3. Do problem 7 of Chapter 7 on page 194.

4. Use the quantity theory of money to answer the following questions.

a) How does a once and for all increase in the supply of money by 10% affect the price level? Does it affect the rate of inflation?

b) How does an increase in the growth rate of the supply of money by 10% affect the inflation rate?

c) What is the inflation rate if the money supply grows at 4% and GDP grows at 2%? How does an increase in GDP growth affect the inflation rate?

d) How do you think an increase in the growth rate of the money supply by 10% affects the real and nominal rates of interest? What is the relationship you use to answer this question?

5. Read the article by Thomas Piketty and Emmanuel Saez, "The Evolution of Top Incomes: A Historical and International Perspective," on the course website. Write down a few questions that occurred to you reading this article. These could be questions you would like to see explored by economists, or they could be things that confused you reading the article, or about what is happening and what its significance might be. (You need not write a lot.)