DUBINS VEHICLE TRACKING OF A TARGET WITH UNPREDICTABLE TRAJECTORY

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ABSTRACT

Motivated by a fixed-speed, fixed-altitude Unmanned Aerial Vehicle (UAV), we seek to control the turning rate of a planar Dubins vehicle that tracks an unpredictable target at a nominal standoff distance. To account for all realizations of the uncertain target kinematics, we model the target motion as a planar random walk. A Bellman equation and an approximating Markov chain that is consistent with the stochastic kinematics is used to compute an optimal control policy that minimizes the expected value of a cost function based on the nominal distance. Our results illustrate that the control can further be applied to a class of continuous, smooth trajectories with no need for further computation.

NOMENCLATURE

r	Dubins-target standoff distance
d	Nominal distance
φ	Viewing angle
σ	Target noise intensity
$\beta(r)$	Distance-dependent discount factor
$u \in \mathcal{U}$	Admissible turning-rate control
dw	Increment of unit intensity Wiener process
$V(r,\varphi)$	Value function

INTRODUCTION

The use of Unmanned Aerial Vehicles (UAVs) to track, protect, or provide surveillance of a ground-based target has recently been the focus of much attention and research. The UAV is assumed to fly at a constant altitude and with a bounded turning radius. This behavior is modeled by a planar Dubins vehicle [1], which gives a good approximation for feasible UAV trajectories, even for realistic UAV kinematics [2]. Our goal in this work is to develop a control policy that allows the Dubins vehicle to maintain a nominal standoff distance from the target without the knowledge of the future target trajectory.

Path planning and shortest path problems in time and space for Dubins vehicles have been studied previously, in [3–6], among others. This type of vehicle has further been studied for the tracking, geolocation, and coverage time of one or more targets by one or more UAVs [7–12].

Ding et. al. [7, 8] employ Pontryagin's principle to plan optimal paths that maximize the Dubins vehicle coverage time of a ground target that is stationary or moving in a straight line. Along similar lines, Quintero et. al. [11] use dynamic programming to develop a control policy that minimizes the geolocation error covariance of the observations of a target that is again assumed to move in a straight line. In both of these works, the control is computed with respect to the current distance between the Dubins vehicle and the target. Although this distance is a relative coordinate, the associated cost function provides a control that is optimal over a planning horizon *for that specific target trajectory only*. Tracking and surveillance of an unknown trajectory would require a separate computation for each possible trajectory, which we avoid with the work presented herein.

Instead, the future motion of the target from its current position is assumed to be a planar random walk [13]. The Dubins vehicle tracking control based on this assumption accounts for wild target kinematics, perhaps that of a target avoiding pursuit. Moreover, any continuous, smooth target trajectory can be considered as a realization of the random walk, although the probability of a particular realization may be very small. Therefore,

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although strictly speaking our tracking control is optimal in the expected value sense for the random walk, it can be applied to a wider class of continuous and smooth target trajectories.

The target speed is included in the form of the intensity of the noise characterizing the random walk. To clarify results we assume a constant noise intensity, although the target speed is not necessarily required to be constant, as described below.

The optimal control feedback policy minimizes the expected value of a cost function which depends on the nominal distance between the Dubins vehicle and the target. The policy is computed off-line using a Bellman equation and an approximating Markov chain that is consistent with the stochastic kinematics [14].

Stochastic problems in the control of Dubins vehicles typically concentrate on variants of the Traveling Salesperson Problem and other routing problems, in which the target location is unknown or randomly-generated [15–17]. Other works examine control methods that direct Dubins vehicle motion toward the maximum of a scalar field [18, 19]. To the best of our knowledge, our work is the first use of stochastic optimal control for the type of problem at hand.

In what follows, we first formulate our problem for the case of a Brownian target and then derive the corresponding relative kinematics model for this problem. Next, the dynamic programming methodologies to compute the optimal control for this problem are provided, and the effectiveness of this approach is demonstrated for Brownian targets and targets with unknown trajectory. We conclude with a discussion and direction for future research.

PROBLEM FORMULATION

We consider a UAV flying at a constant altitude in the vicinity of a ground-based target, tasked with maintaining a nominal distance from the target. The target is located at position $\vec{r}_T(t) = [x_T(t),\, y_T(t)]^T$ at the time point t (see Fig. 1), and since we do not account for the possibility of antagonistic target trajectories, no knowledge of the UAV kinematics or state is assumed. The UAV, located at position $\vec{r}_A(t) = [x_A(t),\, y_A(t)]^T$, moves in the direction of its heading angle θ at a constant speed v_A. The turning rate is determined by a non-anticipative [14], bounded control $u(t) \in \mathcal{U} \equiv \{u : |u| \le u_{\max}\}$, which has to be found. Note that by considering a model with more constraints (e.g., fixed v_A and altitude, single integrator) tracking is more difficult for the UAV.

In our problem formulation, the target motion is unknown. Drawing from the field of estimation, the simplest signal that can be used to describe an unknown model suggests that the motion of the target should be described by a 2D Brownian particle:

$$dx_T(t) = \sigma dw_x, \qquad dy_T(t) = \sigma dw_y \tag{1}$$

where dw_x and dw_y are increments of unit intensity Wiener processes along the x and y axes, respectively, which are mutually

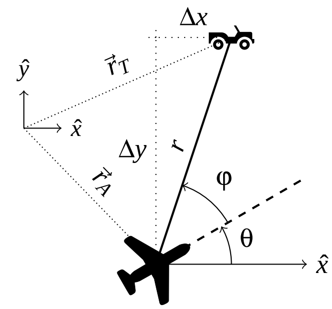

Figure 1. DIAGRAM OF DUBINS VEHICLE AT POSITION $\vec{r}_A$ THAT IS MOVING AT HEADING ANGLE θ AND TRACKING A RANDOMLY-MOVING TARGET AT $\vec{r}_T$ WITH DISTANCE $r = |\vec{r}_A - \vec{r}_T|$ AND RELATIVE ANGLE φ.

independent. The level of noise intensity σ determining the target motion is assumed to be in a range that allows the Dubins vehicle to effectively pursue or maintain pace with the target with high probability. For simplicity, any additional noise due to observation error is also incorporated into the random walk, i.e., the parameter σ.

The UAV can be modeled as a planar Dubins vehicle [1]:

$$\begin{aligned} dx_A(t) &= v_A \cos(\theta(t))\, dt \\ dy_A(t) &= v_A \sin(\theta(t))\, dt \\ d\theta(t) &= -u(t)dt, \qquad u \in \mathcal{U} \end{aligned} \tag{2}$$

where, without loss of generality, we have chosen the sign of $d\theta(t)$ to clarify later results.

In order for the control to be independent of the heading angle of the Dubins vehicle or the absolute position of the Dubins vehicle or target, we relate the problem to relative dynamics based on a time-varying coordinate system aligned with the direction of the Dubins vehicle velocity. The reduced system state is composed of the distance between the Dubins vehicle and target $r = |\vec{r}_T - \vec{r}_A|$ and the viewing angle φ between the Dubins vehicle's direction of motion and the vector from the Dubins vehicle to the target, as seen in Fig. 1:

$$r = \sqrt{(\Delta x)^2 + (\Delta y)^2}, \qquad \varphi = \tan^{-1}\left(\frac{\Delta y}{\Delta x}\right). \tag{3}$$

The combined Dubins-target system (1-3) should maintain the relative distance r at the nominal distance d for all times. To this end, we seek to minimize the expectation of an infinite-horizon cost function $W(\cdot)$ with a distance-dependent discounting factor $\beta(r) > 0$ and with penalty ε (which may be zero) for

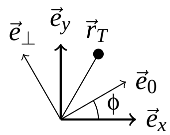

Figure 2. DIAGRAM OF COORDINATE FRAMES

control:

$$W(r,\varphi,u) = E_r^u \left\{ \int_0^\infty e^{-\int_0^t \beta(r(s))ds} k(r(t),u)\, dt \right\} \tag{4}$$
$$k(r,u) = (r-d)^2 + \varepsilon u^2.$$

A high value of $\beta(r)$ along a trajectory places more weight on the instantaneous cost, while $\beta(r)$ near 0 considers future costs, as well as instantaneous cost. If $r \gg d$, the Dubins vehicle must achieve the nominal distance in a future time, while if $r \approx d$, it must simply maintain this distance at the current time. Consequently, we choose $\beta(r)$ as

$$\beta(r) = (\gamma_1 - \gamma_2)\left(\frac{r-d}{d-r_{\min}}\right)^2 + \gamma_2 \tag{5}$$

for positive parameters $0 < \gamma_1 < \gamma_2$, which also yields a greater computational speed than with $\beta(r) = \gamma_1$. The values of γ_1 and γ_2 are such that $\beta(r) > 0$ for any $r \in (r_{\min}, r_{\max})$ for which we compute the optimal feedback control. The bounds of this domain in r can be interpreted as the minimum distance the Dubins vehicle can keep from a stationary target and its maximum observation range. Although in a scenario of this type, control is cheap, the possibility for a non-zero ε is included to compare performance against $\varepsilon = 0$ for a Brownian target.

RELATIVE DUBINS-TARGET KINEMATICS MODEL

Before proceeding with the derivation of the Dubins-target kinematics, we first clarify how rotating the coordinate frame by an angle ϕ affects the target motion model (1). The target position vector is $\vec{r}_T = x_T\vec{e}_x + y_T\vec{e}_y$, where $\vec{e}_x$ and $\vec{e}_y$ are unit vectors along the x and y axis, respectively (see Fig. 2). The same vector described as a backward rotation from the new coordinate system in $(\vec{e}_0, \vec{e}_\perp)$ is

$$\vec{r}_T = x_T \underbrace{(\cos\phi\vec{e}_0 + \sin\phi\vec{e}_\perp)}_{\vec{e}_x} + y_T \underbrace{(\sin\phi\vec{e}_0 - \cos\phi\vec{e}_\perp)}_{\vec{e}_y}. \tag{6}$$

Using Itô calculus and the model (1), the differential change of the target position vector obeys

$$d\vec{r}_T = \sigma dw_0\vec{e}_0 + \sigma dw_\perp\vec{e}_\perp, \tag{7}$$

where

$$dw_0 = dw_x\cos\phi - dw_y\sin\phi \tag{8}$$
$$dw_\perp = dw_x\sin\phi + dw_y\cos\phi. \tag{9}$$

Defining the vector $[dw_0,\, dw_\perp]^T$ and using the last relation, we can easily show that [20]

$$\begin{bmatrix} dw_0 \\ dw_\perp \end{bmatrix}\begin{bmatrix} dw_0 \\ dw_\perp \end{bmatrix}^T = \begin{bmatrix} dt & 0 \\ 0 & dt \end{bmatrix}, \tag{10}$$

which implies that dw_0 and $dw_\perp$ are mutually independent increments of unit intensity Wiener processes [20] along axes of the rotated coordinate system. This is a consequence of the assumption that the components of the original 2D random walk model have the same intensity.

Next, taking into account the random motion of the target, the evolution of $r = |\vec{r}| = |\vec{r}_T - \vec{r}_A|$ can be derived by first considering the differential components $d(\Delta x)$ and $d(\Delta y)$ of the distance vector $\vec{r}$ along the x and y axes:

$$\begin{aligned} d(\Delta x) &= \sigma dw_x - v_A\cos(\theta)\, dt \\ d(\Delta y) &= \sigma dw_y - v_A\sin(\theta)\, dt \end{aligned} \tag{11}$$

According to Itô's Lemma [20], the differential dr is

$$\begin{aligned} dr = {} & \frac{\Delta x}{r} d(\Delta x) + \frac{\Delta y}{r} d(\Delta y) \\ & + \frac{1}{2}\left(\frac{1}{r} - \frac{(\Delta x)^2}{r^3}\right)(d(\Delta x))^2 \\ & + \frac{1}{2}\left(\frac{1}{r} - \frac{(\Delta y)^2}{r^3}\right)(d(\Delta y))^2 \\ & - \frac{(\Delta x)(\Delta y)}{r^3}(d(\Delta x))(d(\Delta y)). \end{aligned} \tag{12}$$

With substitution of partial derivatives and by taking into account that $\cos(\theta + \varphi) = \Delta x/r$ and $\sin(\theta+\varphi) = \Delta y/r$, it can be shown that

$$\begin{aligned} dr = {} & \left(-v_A\cos\varphi + \frac{\sigma^2}{2r}\right) dt \\ & + \sigma\cos(\theta+\varphi)\, dw_x + \sigma\sin(\theta+\varphi)\, dw_y. \end{aligned} \tag{13}$$

Similarly, if we express $(\theta+\varphi) = \tan^{-1}(\Delta y/\Delta x)$,

$$\begin{aligned} d(\theta+\varphi) &= -\frac{\Delta y}{r^2} d(\Delta x) + \frac{\Delta x}{r^2} d(\Delta y) \\ &\quad + \frac{(\Delta x)(\Delta y)}{r^4}(d(\Delta x))^2 - \frac{(\Delta x)(\Delta y)}{r^4}(d(\Delta y))^2 \\ &= \frac{v_A}{r}\sin\varphi dt \\ &\quad -\frac{\sigma}{r}\sin(\theta+\varphi)\,dw_x + \frac{\sigma}{r}\cos(\theta+\varphi)\,dw_y. \quad (14) \end{aligned}$$

Recalling that our coordinate frame is aligned with the direction of Dubins vehicle motion ($\theta = 0$) and that $d\theta = -udt$ (2), the relative Dubins-target kinematics model is

$$dr = \left(-v_A\cos\varphi + \frac{\sigma^2}{2r}\right)dt + \sigma dw_0 \quad (15)$$

$$d\varphi = \left(\frac{v_A}{r}\sin\varphi + u\right)dt + \frac{\sigma}{r}dw_\perp, \quad (16)$$

where dw_0 and $dw_\perp$ are mutually independent increments of unit intensity Wiener processes, in accordance with (8) and (9). Note the presence of a positive bias $\sigma^2/2r$ in the relation for $r(t)$, which is a consequence of the random process included in our analysis.

To determine the set of admissible control $\mathcal{U}$, we look to the limit of no noise ($\sigma = 0$), where the distance and angle differentials are described by

$$dr = -v_A\cos\varphi dt, \qquad d\varphi = \left(\frac{v_A}{r}\sin\varphi + u\right)dt.$$

The steady state will then be defined by

$$0 = dr \Rightarrow \varphi = \pm\frac{\pi}{2} \quad (17)$$

$$0 = \varphi \Rightarrow u = \pm\frac{v_A}{r}. \quad (18)$$

Having in mind the Dubins vehicle's minimum observation distance $r_{\min}$, the admissible set is given by $|u| \le v_A/r_{\min} \equiv u_{\max}$ for $d \in (r_{\min}, r_{\max})$.

MARKOV CHAIN APPROXIMATION AND VALUE ITERATION

When discretizing a state space for dynamic programming in stochastic problems, it is often the case that the chosen spatial and temportal step sizes do not accurately scale in the same way as the stochastic process. To take this into account, we employ the *Markov chain approximation method* [14] for numerically determining the optimal control policy corresponding to the controlled diffusion process (15-16) and cost function (4).

This well-accepted technique involves the careful construction of a discrete-time and discrete-state approximation in the form of a controlled Markov chain that is "locally-consistent" with the process under control. Likewise, an appropriate approximation to the cost function $W(\cdot)$ is chosen by one of several procedures. We will explain the construction in terms of finite difference methods, but note that several other interpretations exist.

For simplicity and to follow the notation of [14], we will write the discrete version of the current system state as $\mathbf{x} = [r,\ \varphi]^T$. Denote $\mathcal{L}^u$ the differential operator associated with the stochastic process (15-16), which, for the sake of brevity, we write in terms of the mean drift $\mathbf{b}(\mathbf{x}) \in \mathbb{R}^2$ and diffusion $\mathbf{a}(\mathbf{x}) \in \mathbb{R}^{2\times 2}$:

$$\mathcal{L}^u = \sum_{i=1}^{2} b_i(\mathbf{x})\frac{\partial}{\partial x_i} + \frac{1}{2}\sum_{i,j=1}^{2} a_{ij}(\mathbf{x})\frac{\partial^2}{\partial x_i \partial x_j}. \quad (19)$$

It can be shown [14] that a sufficiently smooth $W(\mathbf{x},u)$ given by (4) satisfies

$$\mathcal{L}^u W(\mathbf{x},u) - \beta(\mathbf{x})W(\mathbf{x},u) + k(\mathbf{x},u) = 0. \quad (20)$$

The Bellman equation for the minimum cost $V(\mathbf{x})$ over all control sequences is simply

$$\inf_u \left[\mathcal{L}^u V(\mathbf{x}) - \beta(\mathbf{x})V(\mathbf{x}) + k(\mathbf{x},u)\right] = 0, \quad (21)$$

with reflecting boundary conditions $(\nabla V(\mathbf{x}))^T \hat{n} = 0$ at the domain boundaries with normals $\hat{n}$.

The Markov transition probabilities $p(\mathbf{x} \mid \mathbf{y}, u)$ from the state $\mathbf{x}$ to the state $\mathbf{y} \in \mathbb{R}^2$ under the control u appear as coefficients in the finite-difference approximations of the operator $\mathcal{L}^u$ in (20). Using standard approximations for derivatives, the finite-difference discretization for $W(\cdot)$ with step sizes Δr and $\Delta\varphi$ is

$$\begin{aligned} W(r,\varphi,u) = &\ p(r,\varphi \mid r+\Delta r,\varphi)\,W(r+\Delta r,\varphi) \\ &+ p(r,\varphi \mid r-\Delta r,\varphi)\,W(r-\Delta r,\varphi) \\ &+ p(r,\varphi \mid r,\varphi+\Delta\varphi,u)\,W(r,\varphi+\Delta\varphi) \\ &+ p(r,\varphi \mid r,\varphi-\Delta\varphi,u)\,W(r,\varphi-\Delta\varphi) \\ &+ \Delta t_u k(r,u) \quad (22) \end{aligned}$$

where the coefficients of $W(\cdot)$ are the respective transition probabilities given by

$$
\begin{aligned}
p(r,\varphi \mid r+\Delta r,\varphi) &= \Delta t_u \left(\frac{H\left(-v_A\cos\varphi + \frac{\sigma^2}{2r}\right)}{\Delta r} + \frac{\sigma^2}{2(\Delta r)^2} \right) \\
p(r,\varphi \mid r-\Delta r,\varphi) &= \Delta t_u \left(\frac{H\left(v_A\cos\varphi - \frac{\sigma^2}{2r}\right)}{\Delta r} + \frac{\sigma^2}{2(\Delta r)^2} \right) \\
p(r,\varphi \mid r,\varphi+\Delta\varphi,u) &= \Delta t_u \left(\frac{H\left(\frac{v_A}{r}\sin\varphi + u\right)}{\Delta\varphi} + \frac{\sigma^2}{2(r\Delta\varphi)^2} \right) \\
p(r,\varphi \mid r,\varphi-\Delta\varphi,u) &= \Delta t_u \left(\frac{H\left(-\frac{v_A}{r}\sin\varphi - u\right)}{\Delta\varphi} + \frac{\sigma^2}{2(r\Delta\varphi)^2} \right) \\
H(x) &= \max[0,x]. \qquad (23)
\end{aligned}
$$

and Δt_u is an interpolation interval of the piece-wise constant chain, given by

$$
\Delta t_u = \left[\beta + \frac{\left|-v_A\cos\varphi + \frac{\sigma^2}{2r}\right|}{\Delta r} + \frac{\left|\frac{v_A}{r}\sin\varphi + u\right|}{\Delta\varphi} + \frac{\sigma^2}{(\Delta r)^2} + \frac{\sigma^2}{(r\Delta\varphi)^2} \right]^{-1}
$$

Here the chain satisfies the requirement of "local consistency," in the sense that the drift and covariance of the cost function are consistent with the drift and covariance of the cost function associated with original process. The recursive dynamic programming equation for value iteration on the cost function is then

$$
V(\mathbf{x}) = \min_{u\in\mathcal{U}} \left\{ k(\mathbf{x},u)\Delta t(\mathbf{x},u) + \sum_{\mathbf{y}} e^{-\beta\Delta t(\mathbf{x},u)} p(\mathbf{x}\mid\mathbf{y},u)V(\mathbf{y}) \right\} \qquad (24)
$$

for $\mathbf{x}$ inside the computational domain. The domain boundary is reflective, and for the states in this boundary, we use ([14], pp. 143), instead of (24):

$$
V(\mathbf{x}) = \sum_{\mathbf{y}} p(\mathbf{x}\mid\mathbf{y})V(\mathbf{y}). \qquad (25)
$$

Equations (24-25) are used in the standard method of value iteration until the cost converges. From this, we obtain the optimal angular velocity of the Dubins vehicle for any relative distance r and viewing angle φ. Under a sufficient control and reasonable target noise intensity, the Dubins vehicle will remain within a region centered about $r = d$. To this end, we restrict our attention to the semi-periodic computational domain $\mathbf{x} \in (r_{\min}, 2d - r_{\min}) \times [-\pi,\pi)$ discretized into a square grid with spacing $(\Delta r, \Delta\varphi)$. In the examples, the control is obtained by interpolating the current system state to the discretized control $u(r,\varphi)$.

RESULTS

Here we provide a description of the control computed by the dynamic programming methods of the previous section. In simulations, it is shown to be effective in maintaining the relative distance $r = d$ from the Dubins vehicle to the target. In the first of three scenarios, the Dubins vehicle tracks a Brownian target as we have previously discussed. Then, in the second and third scenarios, the same control is applied to the case where the target travels along continuous, smooth trajectories.

In all examples, the nominal distance is 50 [m], and $r_{\min} =$ 10 [m] and $r_{\max} = 90$ [m]. The minimum discount factor γ_1 is 0.05, and the maximum discount factor $\gamma_2 = 3$. The Dubins vehicle speed is chosen so that the vehicle is capable of keeping pace with the speed of the random target motion with high probability. Since the noise $[dw_x,\ dw_y]^T$ is uncorrelated and normally distributed, the total magnitude of the target speed v_T will be distributed as a Rayleigh PDF:

$$
v_T \sim f(v) = \frac{v}{\sigma^2} e^{-v^2/2\sigma^2} \qquad (26)
$$

with parameter σ. Note that this PDF has mean $\bar{v} = \sigma\sqrt{\frac{\pi}{2}}$. We choose $\sigma = 5$ so that if the Dubins vehicle is facing the direction of target motion at traveling at speed $v_A = 10$ [m/s], the probability P_r of being outrun is

$$
P_r\{v_T > v_A\} = 1 - P_r\{v_T \le v_A\} < 15\% \qquad (27)
$$

We point out that our Dubins vehicle is a nonholonomic vehicle tracking a random target; the time required to align motion with that of the target is a hindrance that no bounded control can overcome. It is therefore less likely that (27) will hold. The positive bias in $r(t)$ will augment this effect.

Control Policy

If $\varepsilon = 0$, there is no penalty for control, and the optimal turning rate for the Dubins vehicle as computed by the methods of the previous section is given by a bang-bang controller $u(r,\varphi) \in \{-u_{\max}, u_{\max}\}$, as seen by the thick lines in Fig. 3 for $\sigma = 5$. As such, it is highly responsive to the random motion of the target, and based on previous works, this type of controller is not unexpected [1], [21]. If $\varepsilon = 1$, the penalty gradually smoothes the transitions among the regions. We have labeled several locations on the policy to help indicate the salient features of the control profile. The lines that compose the boundaries of the two control regions may be interpreted as an optimal path of the Dubins-target system in state space, assuming that the initial system state belongs to the lines. The original sign of $d\theta = -u dt$ was chosen so that the path in this state space in some cases reflects the actual path of the Dubins vehicle. Open regions away from these boundaries have the effect of directing the state back to the lines. If the vehicle is in state $(80\,[\text{m}], -\pi/30)$, for example, it is far from the target and directed toward it but with

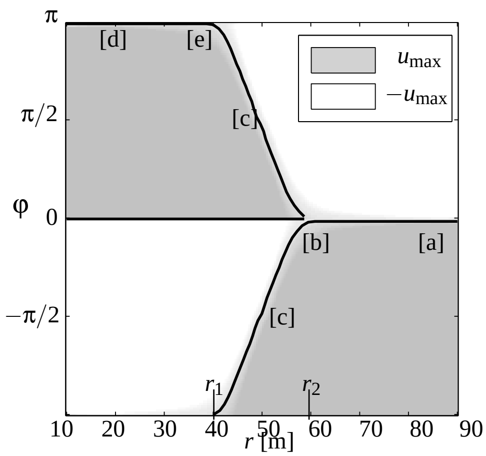

Figure 3. OPTIMAL CONTROL BASED ON DISTANCE TO THE TARGET r AND VIEWING ANGLE φ FOR $\sigma = 5$, $\varepsilon = 1$. BLACK LINES CORRESPOND TO THE CONTROL BOUNDARIES WHEN $\varepsilon = 0$. INDICATED POINTS ARE [a] DUBINS VEHICLE HEADING TOWARD TARGET AT A SMALL ANGLE DISPLACEMENT FROM A DIRECT LINE, [b] START OF CLOCKWISE ROTATION ABOUT TARGET, [c] STEADY STATES AT $\left(d, \pm\cos^{-1}\left(\sigma^2/100v_A\right)\right)$, [d] DUBINS VEHICLE HEADING DIRECTLY AWAY FROM TARGET AT A SMALL ANGLE DISPLACEMENT AWAY FROM A DIRECT LINE, [e] START OF COUNTER-CLOCKWISE ROATION ABOUT TARGET. THE MIDDLE OF THE OPEN REGIONS DIRECT THE DUBINS VEHICLE TO TURN LEFT (WHITE) OR RIGHT (GRAY) TO REACH THE BOUNDING LINES.

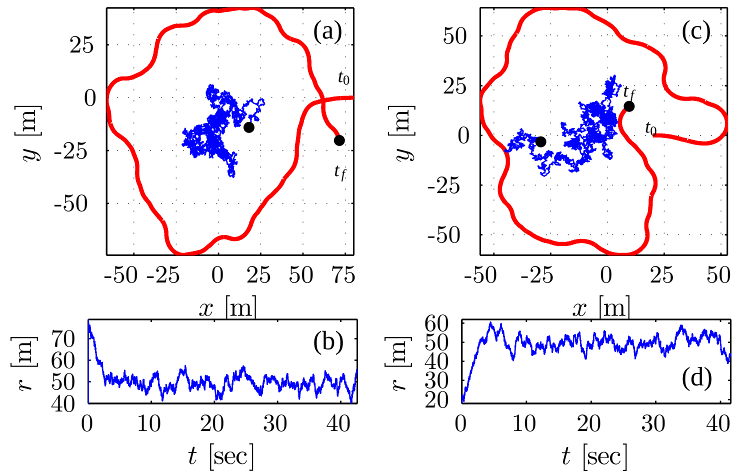

Figure 4. A DUBINS VEHICLE (RED) TRACKING A BROWNIAN TARGET (BLUE). (a) FOR $\varepsilon = 0$, THE DUBINS VEHICLE MUST APPROACH THE TARGET ($t_0 = 0$) BEFORE ENTERING INTO A CLOCKWISE CIRCULAR PATTERN ($t_f = 42.6$) AND (b) ITS ASSOCIATED DISTANCE r ($mean(r) = 50.23$, $std(r) = 4.97$). FOR $\varepsilon = 1$, THE DUBINS VEHICLE TRAJECTORY RESEMBLES THE ORIGINAL, BUT WITH $mean(r) = 49.03$, $std(r) = 5.02$ (c) THE DUBINS VEHICLE BEGINS NEAR THE TARGET ($t_0 = 0$) AND MUST FIRST AVOID THE TARGET using $\varepsilon = 0$ BEFORE BEGINNING TO CIRCLE ($t_f = 41.6$). THE POSITION OF THE TARGET JUMPS SHARPLY EAST NEAR THE END OF THE SIMULATION, AND THE DUBINS VEHICLE IS SHOWN TURNING RIGHT TO AVOID IT. (d) THE ASSOCIATED DISTANCE ($mean(r) = 48.8$, $std(r) = 5.98$). FOR $\varepsilon = 1$, $mean(r) = 48.7$, $std(r) = 5.94$.

a small angular offset that hints at the future rotation about the target. As it approaches the target, the curvature of the region gradually directs the vehicle into a clockwise circle about the target, beginning at $r = r_2$. This continues until the vehicle reaches the steady pattern. Likewise, when the vehicle is moving directly away from the target, it begins to arc toward the steady state configuration at $r = r_1$ as it follows the curvature near the outer boundaries of the figure. It should be noted that the points $(50, \pm\pi/2)$ are found inside the control regions and not on the boundary due to the bias in $r(t)$.

In simulations, the actual evolution of the state (r, φ) is not smooth due to the random motion of the target. The Dubins vehicle spends more time in the open regions attempting to return to the lines than it does on the lines; this does not, however, detract from the trajectory of the Dubins vehicle as the control is able to compensate. Since we began this problem with an infinite-horizon cost, the control is highly robust to such deviations, as we will show in simulations below.

The positioning of r_1 and r_2 determine the state at which the Dubins vehicle transitions from the act of "avoiding" or "chasing" the target when it is too close or too far, respectively, into the act of maintaining the distance d. Owing to the bias in the mean drift of dr (15), $|r_1 - d| > |r_2 - d|$, eg. a Dubins vehicle avoiding the target must expect the bias to "help" with this action, while when chasing, the bias might hinder these efforts.

Scenarios

We show implementations of this control for a Brownian target when the Dubins vehicle is initially too far from the target in Fig. 4(a-b) and when it is initially too close to the target in Fig. 4(c-d). A variety of behaviors is observed, but the associated costs remain small. Since a small time-step was used for simulation, the Dubins vehicle trajectory using the control penalty $\varepsilon = 0$ was indistinguishable from when $\varepsilon = 1$, but since its response is less sensitive to small displacements in target location when $\varepsilon = 1$, the average standoff distance was slightly affected.

To emphasize the level of robustness provided by the control, the original assumption that the target position evolves as a 2D random walk is dropped. We exhibit the response to a target moving in the direction of its heading angle θ_T with velocity v_T:

$$dx_T(t) = v_T \cos\theta_T dt, \qquad dy_T(t) = v_T \sin\theta_T dt$$

where $v_T = 5\sqrt{2/\pi} \approx 4$ [m/s] is based on the noise intensity for which control has previously been computed, and the evolution of θ_T is chosen differently for each scenario below.

In the first scenario, we fix the target to move in the East direction ($\theta_T = 0$) in order to compare against several other papers dealing with the same target trajectory. Since target motion is

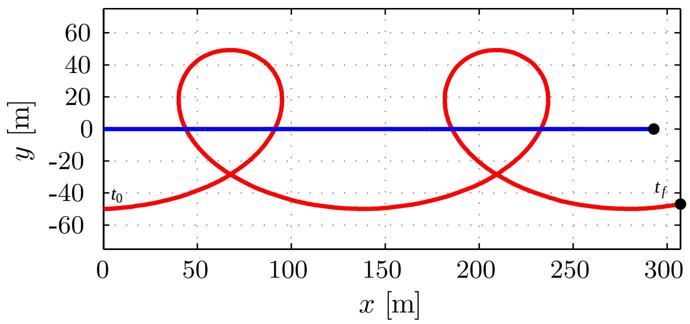

Figure 5. A DUBINS VEHICLE (RED) TRACKING A TARGET (BLUE) MOVING DUE EAST USING THE CONTROL FOR A BROWNIAN TARGET ($\varepsilon = 0$). THE DUBINS VEHICLE PASSES OVER THE TARGET TRAJECTORY IN ENLONGATED CIRCLES, $t \in [0, 73.4]$. $mean(r) = 49.7$ [m], $std(r) = 1.39$. FOR $\varepsilon = 1$, $mean(r) = 49.9$ [m], $std(r) = 2.19$.

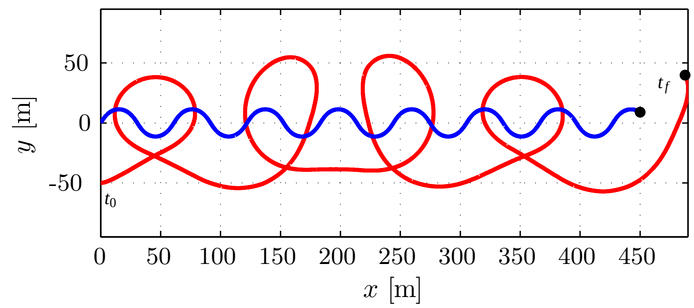

Figure 6. A DUBINS VEHICLE (RED) TRACKING A TARGET (BLUE) MOVING IN A SINUSOIDAL PATH USING THE CONTROL FOR A BROWNIAN TARGET ($\varepsilon = 0$). THE DUBINS VEHICLE CIRCLES THE TARGET IN ECCENTRIC CIRCLES WHOS SHAPE IS DETERMINED BY THE CURRENT POSITION OF THE TARGET ALONG ITS TRAJECTORY, $t \in [0, 147.0]$. $mean(r) = 49.6$ [m], $std(r) = 1.37$. FOR $\varepsilon = 1$, $mean(r) = 49.8$ [m], $std(r) = 2.24$.

no longer random, our control is no longer strictly optimal. It is seen in Fig. 5, however, that the vehicle remains near the nominal standoff distance, with the average distance slightly below d in each case since the noise bias in $r(t)$ is no longer present.

In a second example, a sinusoidal path heading East ($\theta_T(t) = \cos(\pi t/10)$) now describes the target motion. The performance of the Dubins vehicle tracking this path is seen in Fig. 6.

In Fig. 7 we provide a comparison of the control algorithm's performance against target trajectories under various target kinematics. Fig. 7(a) shows the mean cost of $(r(t) - d)^2$, further averaged over 100 simulations, where the target exhibits a random turning rate with intensity σ_θ:

$$d\theta_T = \sigma_\theta dw. \tag{28}$$

Fig. 7(b) shows the same cost for deterministic, sinusoidal target trajectories with varying frequency:

$$d\theta_T = -\omega \sin(\omega t)\, dt. \tag{29}$$

In both cases, it is seen that the control algorithm's performance degrades as the speed of the target increases beyond the anticipated speed of $5\sqrt{2/\pi}$, which is to be expected. However, the cost is relatively homogeneous for a wide range of target noise intensities, and the algorithm performance for a sinsoidal trajectory is similar to that of a random walk.

CONCLUSIONS AND FUTURE WORK

This paper studies the problem of a UAV flying with fixed speed attempting to maintain a nominal distance from a ground-based target with an unknown trajectory. In particular, a model of a Brownian particle is assumed for the target, which leads to a novel Dubins problem for tracking a stochastic target. We employ the Markov chain approximation method to determine the optimal turning rate of the Dubins vehicle based on a relative distance and viewing angle to the target. A Markov chain approximation that is locally-consistent with the system under control is constructed on a discrete state-space, and value iteration on the associated cost function produces a control to minimize the mean squared distance of the target in excess of a nominal distance. The computed control is shown to be effective in tracking the target from any initial condition and is able to anticipate the tendency for the target to drift away from the Dubins vehicle.

The biggest shortcoming in the formulation is due to the time required for the UAV to change its heading angle to account for unexpected target motion. If a target unexpectedly advances in proximity, the Dubins vehicle must have a relatively large velocity and turning rate to dodge it. This possibility is central to the uninformative property of the problem, but can be avoided if the target motion can be locally predicted. Should the target also be modeled as a Dubins vehicle with a Brownian heading angle, the knowledge of the target's heading angle would provide the Dubins vehicle with an indicator of its immediate motion and an appropriate response (control) to this prediction.

The assumption of random target dynamics coupled with an infinite horizon cost function has the fundamental advantage of creating a highly robust control. Provided the target speed (and, therefore, intensity of the directional component noises) is known, a variety of trajectories can be tracked using this methodology, despite the fact that the bias in the Dubins-target distance is no longer present in the case of natural motion. We view our work as evidence to support the practice of substituting deterministic target kinematics with Brownian motion, especially in the case of a UAV tracking scenario, where a typical target may exhibit a rapidly-changing or stochastic trajectory. If the stochastic substitute not only *includes* all possible trajectories, but also *reflects* the distribution of these possible trajectories, the resulting

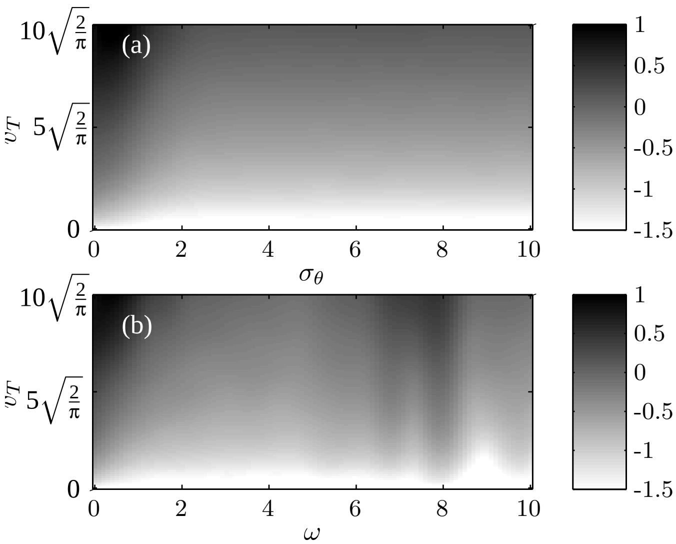

Figure 7. EFFECT OF TARGET SPEED v_T and (a) NOISE INTENSITY σ (28) or (b) SINUSOIDAL FREQUENCY ω (29) ON THE LOG MEAN COST $\log_{10} mean\left\{(r(t)-d)^2\right\}$ OF A 60-SECOND SIMULATION.

control algorithm performs well against a wide variety of targets.

The computational burden for computing the control is not small; however, it needs to only be computed once for a given target noise, regardless of trajectory shape, and takes into account kinematic nonlinearities. Furthermore, since prediction of a global system trajectory in multi-agent systems is often not possible, we believe that this type of work has the potential for extensions to multi-agent tracking.

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