Optimization of Taxiway Traversal at Congested Airports

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Airport runways and taxiways have been identified as a key source of system-wide congestion and delay in the over-strained commercial air traffic system. To combat this growing problem, we present a novel approach for taxiway scheduling and traversal. Aircraft must traverse a taxiway, represented by a graph, from gates to their respective runways and arrive at their scheduled times while adhering to safety separation constraints. We describe a combinatorial mixed integer linear program to simultaneously determine the optimal push-back time windows, aircraft speeds, stopping times, and in particular, traversal paths for a given graph and an imposed flight schedule. Several scenarios are presented to demonstrate possible uses for this tool.

Nomenclature

Δ	Spatial separation requirement (based on weight class)
Δ^T	Temporal runway separation requirement (based on weight class)
$U_{\max}$	Taxiway maximum speed limit
$U_{\min}$	Taxiway minimum speed limit
K	Set of all flights
t_k	Push-back time of departing aircraft k
$t_k^{(\text{sched})}$	Scheduled runway arrival/departure time of aircraft k
t'_k	Perturbed schedule variable of aircraft k
$\mathcal{P}_k$	Set of all possible paths of an aircraft
p	A particular path $\in \mathcal{P}$
d_{ij}	Length of arc (i,j)
f_{ijk}	Reciprocal speed of aircraft k along arc (i,j)
τ_{ijk}	Stopping time of aircraft k along arc (i,j)
z_{ijk}	Binary path decision variable
$B_{k_\alpha k_\beta i}$	Binary order decision variable, $k_\alpha, k_\beta \in K$
κ	Probability of constraint violation
λ	Inverse cumulative distribution of $1-\kappa$
$\sigma_{(\cdot)}$	Shape parameter for distribution of random variable $(\cdot)$
Subscript	
k	Aircraft/flight number, $k \in K$
k_α, k_β	To distinguish between two aircraft

I. Introduction

The consumer demand placed on the commercial air transportation network has increased dramatically in recent years, without an equivalent improvement in airport and airborne traffic capacities.[1] As this trend

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is likely to continue, the potential for delays, financial and environmental inefficiencies, and safety concerns has led to a considerable amount of research to accommodate this growth in both the en-route and airport sectors.[2]

Recent improvements to en-route capabilities have caused a shift in the system bottleneck toward the airport sector. Many airports are already operating at or close to their maximum capacity,[3] causing instances when aircraft must queue on the taxiway while engines continue to operate. Consequently, not only does limited airport capacity create delays, but can also cause unnecessary fuel consumption and air pollution.

In this paper we describe a method to increase an airport's taxiway capacity using an optimal aircraft taxi scheduling, taking into account the structure of taxiway network. Previous works typically use a graph- or network-based approach, and authors implementing this method mainly focus on genetic algorithms,[4,5] mixed-integer linear programs (MILPs), or a hybrid of these.[6,7] The MILP method has proven to be practical for real-time computation and is along the line of the work presented herein. We consider aircraft taxi speeds, push-back times, stopping times, as well as taxiway routes, but unlike previous methods, these taxi scheduling characteristics are all coupled as a part of a single optimization problem. Because of this, we can determine the feasibility of a taxi schedule that includes knowledge of the taxiway topology. This feasibility indicator can be used either to detect conflicting constraints in the taxi schedule, analyze a taxiway traversal policy, or to estimate airport capacity under a variety of conditions, including uncertainty.

Various models using MILP have been explored in literature. Roling and Visser[8] consider a discrete time taxi scheduling problem with constant aircraft taxi velocities. Balakrishnan et al.[9] extend the formulation of Bertsimas[1] to the airport, but constraints against aircraft overtaking one another on the same path are only incorporated after the optimal solution is found. Smeltink et al.[10] developed a MILP which reduces taxiway delays, but did not take into account all forms of taxiway separation constraints. This was fixed by Rathinam et al.,[11] who also significantly reduced the state space of the problem, but still only one path per plane is considered. Marín[12,13] frames the taxiway as a linear multi-commodity flow problem with additional constraints, which again causes all routes between origin and destination nodes to be fixed *a priori* using a shortest-path algorithm.

In what follows, we first introduce in Section II the motivation for our approach, and in Section III we formulate our program. Section IV provides results from several scenarios and describes relative benefits as compared to a baseline First-Come-First-Served (FCFS) policy. Section V summarizes our contributions from this paper and provides directions for future research.

II. Details

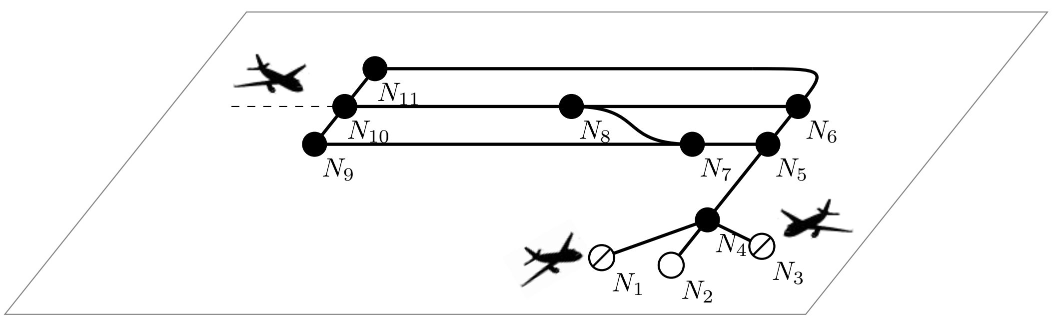

Figure 1. The Airport Taxiway Problem: Solid lines correspond to taxiway arcs and a runway comprised of (N_{10}, N_8) and $(N_8, N-6)$. Nodes (filled circles) are placed at locations of possible path conflicts, and open circles denote gates. See text for explanation.

Figure 1 illustrates a possible scenario in which three aircraft must traverse a taxiway. Two departing aircraft, beginning at gates N_1 and N_3, must reach the node N_{10} at the times $t_1(N_{10})$ and $t_2(N_{10})$, respectively, as given by the imposed departure schedule. Simultaneously, the third aircraft is arriving, and once it has decelerated, it must taxi to gate N_2. Based on this information, we would like to compute the optimal taxi schedule, including *routes, speeds, stopping times* and *gate push-back times*, such that each aircraft quickly reaches its destination.

Clearly, in Figure 1 there are two possible routes of similar length for each aircraft, but there exists a potential for two aircraft to come within a close proximity, or even be in conflict, while traversing the taxiway arcs and nodes shared by their routes. To avoid this, we define both temporal and spatial separation constraints among aircraft, and these must be included into the taxi schedule. A more realistic scenario may include a larger number of aircraft and alternative routes, but in essence the problem of computing the taxi schedule remains the same.

Under such a precise control of aircraft flow, we see the aircraft taxi schedule problem as a continuous-time optimal control problem[14] under geometric constraints of the airport taxiway layout, departure and arrival times, as well as safety separation constraints among aircraft. To deal with a well-formulated optimal control problem, one specifies a cost function, e.g. the minimum total taxi time, and computes the optimal taxi schedule that minimizes this cost function. However, the velocity of each aircraft is a continuous-time function, and computing its time profile under the separation constraints in a continuous-time optimal control framework is a challenging problem. Therefore, it is not surprising that previous papers consider discretization in time, space, or both space and time (see Section I).

Our work is motivated by the fact that the optimal control problem can be significantly simplified *under the assumption that the aircraft taxi speed is constant on a given arc*, although this constant is aircraft- and arc-dependent. Then, spatial safety separation constraints for taxiing aircraft can be written in the form of temporal constraints. This is important because the required safety separation at runaways is most naturally described by the separation in time between aircraft departures and arrivals. Consequently, both spatial and temporal safety separation among aircraft can be formulated as a homogeneous set of temporal separation constraints. This allows us to compute an optimal taxi schedule as the solution of a MILP.

Inefficiencies in aircraft taxiway traversal can result in queues and therefore periods during which aircraft engines are unnecessarily running. To avoid this, we consider a taxi scheduling plan that can be performed with minimal or no stopped aircraft in the interest of fuel consumption. This means that any departure delay imposed by the Federal Aviation Administration (FAA) should ideally be managed either by delaying gate push-back, or if necessary, by distributing the delay along the route by appropriately adjusting speed. However, we do allow for stopping to achieve a degree of freedom requested by human operators.

III. Formulation

Since arrival, departure, and gate docking processes are all interconnected at runways, gates, apron entries, and taxiways, graphical or network-based approaches provide a natural model to explicitly monitor or control coordination among aircraft at the intersections of these locations. We model our taxiway as a directed graph $G(N, A)$ consisting of a set $N = \{N_1, N_2, ...N_n\}$ of n nodes and a set A of m ordered pairs, or arcs, of N, $\{a_1, a_2, ...a_m\}$, such that each arc is of the form (N_i, N_j), which we notate as arc (i, j), $i, j = 1, 2, \ldots, n$. Each direction of a taxiway segment of length d_{ij} corresponds to a single arc $(d_{ij} = d_{ji})$. Nodes consist of gates, runways, aprons, and other points where taxiway paths intersect. The optimal taxi schedule problem for a specified horizon of all flights in the set K can then be formulated as follows:

Variables:
To control speed we introduce a variable f_{ijk} that is a function of the inverse of the velocity $v_k(i, j)$ of aircraft k traveling on arc (i, j).

$$f_{ijk} = 1/v_k(i, j) \tag{1}$$

The variable t_k is the push-back time of aircraft $k \in K$, and the total time an aircraft k stops along arc (i, j) is notated as τ_{ijk}. A binary variable z_{ijk} is 1 if aircraft k travels along arc (i, j) and 0 otherwise, and a binary variable $B_{k_\alpha k_\beta i}$ is 1 if aircraft k_α arrives at node i before aircraft k_β and 0 otherwise $(k_\alpha, k_\beta \in K; k_\alpha \neq k_\beta)$.

Aircraft must arrive at destination node by scheduled time:
We define a path p as a sequence of arcs for an aircraft to follow, and $|p|$ as the number of segments in that sequence. Then for each aircraft k, and for each path p in the set of all possible paths $\mathcal{P}_k$ for aircraft k,

$$t_k + \sum_{(i,j)\in p} (f_{ijk} d_{ij} + \tau_{ijk}) - M \left(|p| - \sum_{(i,j)\in p} z_{ijk} \right) \leq t_k^{(\text{sched})} \tag{2}$$

This constraint is made inactive with the large constant M if the path p is not chosen by aircraft k.

Aircraft path must be continuous:
For each aircraft k, and for each node i in arc (i, j),

$$\sum_j z_{ijk} - \sum_j z_{jik} = \begin{cases} 1 & \text{if } i = \text{(start)} \\ -1 & \text{if } i = \text{(end)} \\ 0 & \text{else} \end{cases} \tag{3}$$

Note that this constraint appears in a simple shortest-path problem. Since for this problem, the objective function does not explicitly minimize the number of traversed arcs, the following constraints must also be applied:

Each node may be visited a maximum of once per aircraft:
For each aircraft k, and for each node i in arc (i, j),

$$\sum_j z_{ijk} \quad \leq \quad 1 \qquad \text{if } i \neq \text{(end)} \tag{4}$$

$$\sum_j z_{jik} \quad = \quad 1 \qquad \text{if } i = \text{(end)} \tag{5}$$

Separation requirements:
Aircraft must be separated by a minimum spatial distance $\Delta_{k_\alpha k_\beta}$ on the taxiway, dependent on the weight classes of the aircraft k_α and k_β, at all times. We can ensure that this is true if three restrictions are imposed. First, when an aircraft arrives at a node, the actual distance between aircraft ℓ is at least equal to $\Delta_{k_\alpha k_\beta}$. Likewise, when one aircraft has departed a node and another trails it, ℓ must still be greater than $\Delta_{k_\alpha k_\beta}$. If two aircraft travel an arc (i, j) with constant (but possibly different) speeds, the separation constraint at i and at j will also be satisfied along the entire arc as long as the order in which aircraft arrive at the preceding node i is the same as the order at the next node j.

We first must restrict the distance ℓ between two aircraft as they approach a common node in both aircraft's paths. If aircraft k_α arrives at node v before aircraft k_β, we require that the distance ℓ be greater than the minimum separation distance, i.e.,

$$B_{k_\alpha k_\beta v} \left(\ell - \Delta_{k_\alpha k_\beta}\right) \geq 0$$

where $\Delta_{k_\alpha k_\beta}$ is an element in a matrix of separation requirements indexed by the weight classes of the leading and following aircraft.

Our program deals mainly with temporal constraints, but here we must consider spatial separation requirements. This is fully explored in Rathinam et al.,[15] but for the situation when the times at each node $t_k(\cdot)$ are known. For our problem, these times are path dependent, and so we extend this analysis to consider all possible paths and arc speeds on these paths. Since aircraft speed on an arc is constant, distance-time relations can relate spatial distances to temporal constraints (see Figure 2).

By enumerating the set of possible paths $\mathcal{P}$ and summing the total time required for each path, we can write the constraint as follows. For each aircraft $k_\alpha \neq k_\beta$, for each node $v \in \mathcal{P}_\alpha \cap \mathcal{P}_\beta$, for each path p_α from the starting node of k_α to v and path p_β from the starting node of k_β and ending in (u, v),

$$\begin{aligned} &\left(t_{k_\alpha} + \sum_{(i,j)\in p_\alpha} f_{ijk_\alpha} d_{ij} + \tau_{ijk_\alpha}\right) - \left(t_{k_\beta} + \sum_{(i,j)\in p_\beta} f_{ijk_\beta} d_{ij} + \tau_{ijk_\beta}\right) + \Delta_{k_\alpha k_\beta} f_{uvk_\beta} \\ &\quad -M\left(|p_\alpha| - \sum_{(i,j)\in p_\alpha} z_{ijk_\alpha}\right) - M\left(|p_\beta| - \sum_{(i,j)\in p_\beta} z_{ijk_\beta}\right) - M\left(1 - B_{k_\alpha k_\beta v}\right) \leq 0 \end{aligned} \tag{6}$$

This constraint is made inactive with the large constant M if the paths p_α and p_β are not chosen, or if aircraft k_α does not arrive at node v before k_β.

We are also interested in the case where two aircraft depart a common node. If aircraft k_α departs at

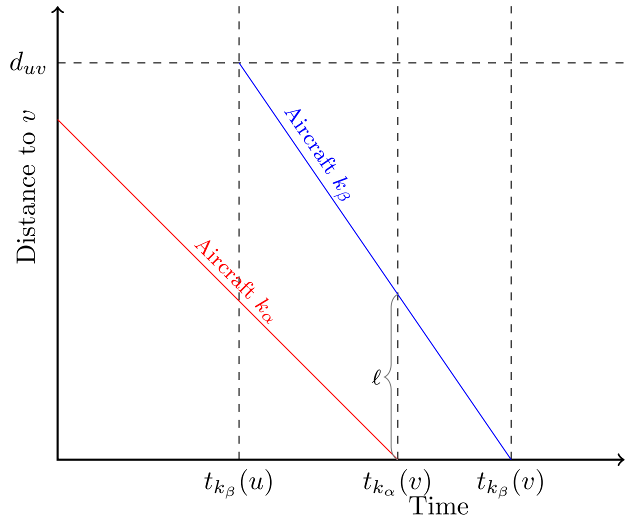

Figure 2. Distance-time relation for two aircraft (k_α and k_β) approaching a common node v (adapted from[11]). From similar triangles, $\ell = \left(\frac{t_{k_\beta}(v)-t_{k_\alpha}(v)}{t_{k_\beta}(v)-t_{k_\beta}(u)}\right) d_{uv}$

node u before aircraft k_β, we require that the distance ℓ be greater than the minimum separation distance for aircraft of weight class $\Delta_{k_\alpha k_\beta}$:

$$B_{k_\alpha k_\beta u}\left(\ell - \Delta_{k_\alpha k_\beta}\right) \geq 0$$

or equivalently (see Figure 3), for each aircraft $k_\alpha \neq k_\beta$, for each node $u \in \mathcal{P}_\alpha \cap \mathcal{P}_\beta$, for each path p_α and path p_β from the aircraft's respective starting positions to u where p_α continues on (u, v), *

$$\begin{aligned} &\left(t_{k_\alpha} + \sum_{(i,j)\in p_\alpha} f_{ijk_\alpha} d_{ij} + \tau_{ijk_\alpha}\right) - \left(t_{k_\beta} + \sum_{(i,j)\in p_\beta} f_{ijk_\beta} d_{ij} + \tau_{ijk_\beta}\right) + \Delta_{k_\alpha k_\beta} f_{uvk_\alpha} \\ &-M\left(|p_\alpha| - \sum_{(i,j)\in p_\alpha} z_{ijk_\alpha}\right) - M\left(|p_\beta| - \sum_{(i,j)\in p_\beta} z_{ijk_\beta}\right) - M\left(1 - B_{k_\alpha k_\beta u}\right) \leq 0 \end{aligned} \tag{7}$$

This constraint is made inactive with the large constant M if the paths p_α and p_β are not chosen, or if aircraft k_α does not arrive at node u before k_β.

Crossing constraints:
We have guaranteed that two aircraft are adequately separated at each node, and keep a constant speed on every arc (i, j). By requiring that the order in which aircraft arrive at i remains the same as the order at j, aircraft will not overtake or cross during arc traversal. For all aircraft $k_\alpha \neq k_\beta$, and all arcs (i, j) or $(j, i) \in \mathcal{P}_{k_\alpha} \cap \mathcal{P}_{k_\beta}$,

$$B_{k_\alpha k_\beta i} - B_{k_\alpha k_\beta j} - \left(2 - z_{ijk_\alpha} - z_{ijk_\beta} - z_{jik_\alpha} - z_{jik_\beta}\right) \leq 0 \tag{8}$$

$$B_{k_\alpha k_\beta i} - B_{k_\alpha k_\beta j} + \left(2 - z_{ijk_\alpha} - z_{ijk_\beta} - z_{jik_\alpha} - z_{jik_\beta}\right) \geq 0 \tag{9}$$

If arc (i, j) is not chosen in any direction by aircrafts k_α and k_β this constraint is inactive; otherwise $B_{k_\alpha k_\beta i} = B_{k_\alpha k_\beta j}$.

One aircraft must arrive at a node before another:

$$B_{k_\alpha k_\beta i} + B_{k_\beta k_\alpha i} = 1 \tag{10}$$

Runway separation:
Due to runway separation requirements, no two aircraft can occupy a runway node for a specified time

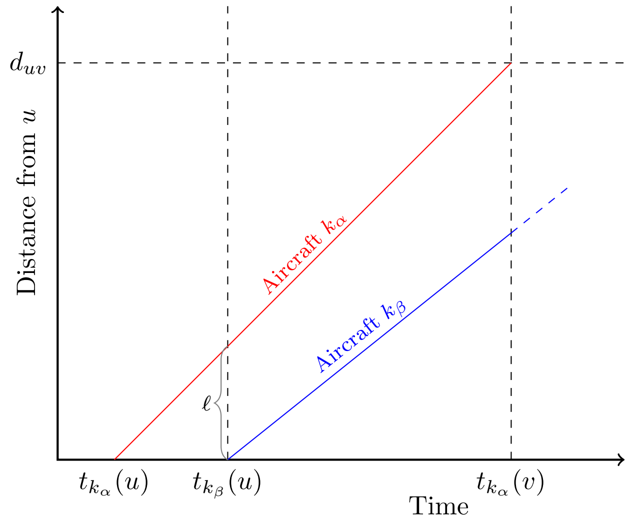

Figure 3. Distance-time relation for two aircraft (k_α and k_β) departing a common node u. Similar triangles show that $\ell = \left(\frac{t_{k_\beta}(u)-t_{k_\alpha}(u)}{t_{k_\alpha}(v)-t_{k_\alpha}(u)}\right) d_{uv}$.

interval after one aircraft departs on that runway. For each aircraft k_α and $k_\beta \neq k_\alpha$, for each runway node r, for each path p_α from the starting node of k_α to its destination node u at the start of the runway containing r, and for each path $p_\beta \in \mathcal{P}_{k_\beta}$ from the starting node of k_β to r,

$$\left(t_{k_\alpha} + \sum_{(i,j)\in p_\alpha} f_{ijk_\alpha} d_{ij} + \tau_{ijk_\alpha}\right) - \left(t_{k_\beta} + \sum_{(i,j)\in p_\beta} f_{ijk_\beta} d_{ij} + \tau_{ijk_\beta}\right)$$
$$-M\left(|p_\alpha| - \sum_{(i,j)\in p_\alpha} z_{ijk_\alpha}\right) - M\left(|p_\beta| - \sum_{(i,j)\in p_\beta} z_{ijk_\beta}\right) - M\left(1 - B_{k_\alpha k_\beta u}\right) \leq -\Delta^T_{k_\alpha} \tag{11}$$

where $\Delta^T_{k_\alpha}$ is the required temporal separation based on the weight class of aircraft k_α.

Aircraft should also avoid taxiing along a runway. To this end, for each departing aircraft k, and for each arc (i,j) along each runway,

$$\begin{aligned} z_{ijk} &= 0 \\ z_{jik} &= 0 \end{aligned} \tag{12}$$

Likewise, the constraint in Equation 12 is imposed for each arriving aircraft k and each arc (i,j) on a runway other than its designated landing runway. Any airport-specific path restrictions can also be implemented with constraints of this form.

Variable bounds:
Variables are constrained to only take on realistic values

$$\begin{aligned} t_k &\geq 0 \\ f_{ijk} &\geq 1/U_{\max} && \text{for departing aircraft \& decelerated landing aircraft} \\ f_{ijk} &\leq 1/U_{\min} \\ \tau_{ijk} &\geq 0 \\ z_{ijk} &= \{0,1\} \\ B_{k_\alpha k_\beta i} &= \{0,1\} \end{aligned} \tag{13}$$

Uncertainty:
Should a pilot or ground traffic controller not adhere to the prescribed scheduling, the optimal taxi solution determined by our formulation must be robust to any small deviations from the solution x. We therefore solve for the mean $\bar{x}$ of random variables x so that the constraints are robust to a known spread about the mean. Here we assume a Gaussian distribution, and other distributions follow similarly.

Uncertainty is assumed to arise in aircraft speeds[a], gate push-back times, and stopping times. We write

$$\begin{aligned} f_{ijk} &\sim \bar{f}_{ijk} + N\left(0, \sigma_f^2\right) \\ t_k &\sim \bar{t}_k + N\left(0, \sigma_t^2\right) \\ \tau_{ijk} &\sim \bar{\tau}_{ijk} + N\left(0, \sigma_\tau^2\right). \end{aligned}$$

Our program constraints (2-13) can be written as $Ax \leq b$, where each constraint $a_j x \leq b_j$ forms a row of coefficients $a_{ij} = [A]_{ij}$. Then all constraints j are made valid by minimizing the probability of constraint violation κ:

$$\begin{aligned} \kappa &\geq \Pr\left(a_j x > b_j\right) \qquad \forall j \\ &= \Pr\left(a_j \bar{x} + N\left(0, \sum_{\text{uncertain } i} a_{ij}^2 \sigma_i^2\right) > b_j\right) \\ &= 1 - \Phi\left(\frac{1}{\sqrt{\sum a_{ij}^2 \sigma_i^2}} \left[b_j - a_j \bar{x}\right]\right). \end{aligned}$$

where $\Phi(\cdot)$ is the chosen cumulative distribution function. To minimize κ, we introduce a variable λ to be maximized:

$$\max \quad \lambda \tag{14}$$

$$\text{s.t.} \quad a_j \bar{x} + \lambda \sqrt{\sum_i a_i^2 \sigma_i^2} \leq b_j \qquad \forall j \tag{15}$$

$$a_j \bar{x} \leq b_j \qquad \forall j \tag{16}$$

where

$$\lambda = \Phi^{-1}\left(1 - \kappa;\ 0, 1\right). \tag{17}$$

Note that the inclusion of λ in the objective function competes with total taxi time minimization (see Section IV). Furthermore, for λ to represent the same value in all constraints, the chosen distribution must have nice properties that allow scale and shape parameters of the cumulative density function of the sum of non-identical r.v.'s to be made independent of the non-zero coefficients in the constraints, e.g., Gaussian and Cauchy distributions. Alternatively, for distributions where this is not possible or difficult (e.g., sum of non-identical uniform distributions), a user-chosen value κ may be included in each constraint from the original formulation that guarantees that feasible solutions are robust with probability $1 - \kappa$, i.e.,

$$a_j \bar{x} + \Phi^{-1}\left(1 - \kappa;\ \mu = 0,\ \sigma^2 = \sum_i a_{ij}^2 \sigma_i^2\right) \leq b_j \qquad \forall j.$$

For separation constraints, these robustness requirements essentially restrict the probability of the intersections of the possible positions of the aircraft.

Minimum perturbed schedule:
In certain circumstances, computing the optimal schedule would be an infeasible problem. This would be a clear indication that the requested plan of departures and arrivals could not be achieved, or that the airport is operating beyond its maximum taxiway capacity. Should any of the preceding constraints cause the program to become infeasible, an operator might wish to re-solve the problem to find a minimally perturbed flight

[a]Note that if reciprocal speed is Gaussian, velocity arises from a distribution with a heavier tail that favors faster speeds

schedule that adheres to all constraints. To this end, we introduce the perturbed schedule variable t'_k for a flight k, and a variable δ_k that represents the absolute difference between the perturbed schedule and original schedule $|t_k - t'_k|$.

$$\begin{aligned} -\delta_k - t'_k &\leq -t_k^{(\text{sched})} \\ -\delta_k + t'_k &\leq t_k^{(\text{sched})} \end{aligned} \tag{18}$$

Then Equation (2) becomes

$$t_k + \sum_{(i,j)\in p} (f_{ijk} d_{ij} + \tau_{ijk}) - M \left(|p| - \sum_{(i,j)\in p} z_{ijk} \right) \leq t'_k \tag{19}$$

and the objective function minimizes $\sum_k \delta_k$. Similar formulations can be used to find a minimally perturbed probability of constraint violation κ.

IV. Results

In this section, we provide results of the implementation of our formulation. As a first scenario, three aircraft traverse a taxiway in the shape of a kite in a minimal amount of time. Aircraft 1 and 2 must switch places at N_2 and N_4, while aircraft 3 must travel from N_1 to N_5 (see Figure 4). This simple arrangement leads to 729 possible path combinations, 3461 constraints, 115 solution points, and 60077 non-zero coefficients. Constraints are automatically generated in Matlab, and the dynamic search feature of CPLEX is used to determine the solution with the minimal total taxi time or minimal probability of constraint violation; this example requires approximately 7 seconds on an Intel i7 with 4GB RAM.

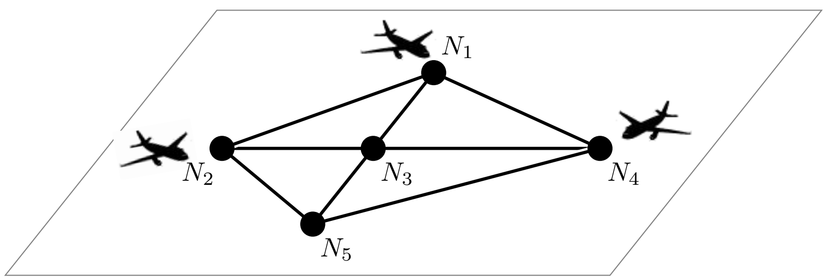

Figure 4. Example taxiway

Figure 5 compares the optimal solutions from our program to those determined by a First-Come-First-Served (FCFS) policy in which priority is given according to the original schedule. In all implementations of our formulation on this problem, the total taxi time required and the probability of constraint violation are less. As the intervals between scheduled flights decrease, i.e., as congestion increases, the benefits as compared to a FCFS policy become more noticeable.

With the choice of many possible paths for each aircraft, minimization of total taxi time is often in direct conflict with minimization of the probability of constraint violation. Therefore we consider the following objective function

$$\max \qquad \eta\lambda \ + \ \theta \sum_k t_k$$

with weight η on λ and weight θ on the total taxi time. For the example in Figure 4, we can enumerate the Pareto outcomes for which any decrease in total taxi requires an increase in the probability of constraint violation. With larger κ, the spacing between flights' trajectories may approach the $\Delta_{k_\alpha k_\beta}$ limit, and more flights can be squeezed in an efficient path, causing the total taxi time to decrease. When flights must be further separated as a precaution based on κ, one or more flights might be required to take a significantly longer path, lengthening the total taxi time.

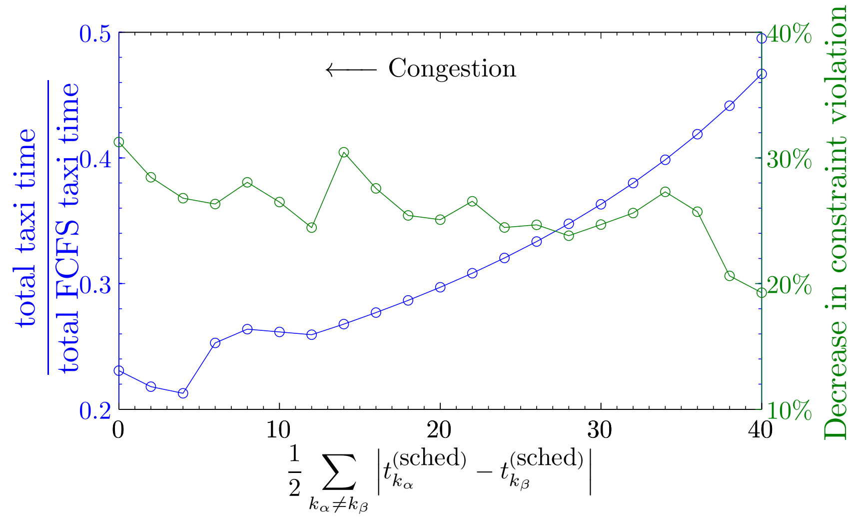

Figure 5. Benefits of formulation compared to FCFS policy. As congestion increases, advantages in total taxi time and probability of constraint violation are more noticeable.

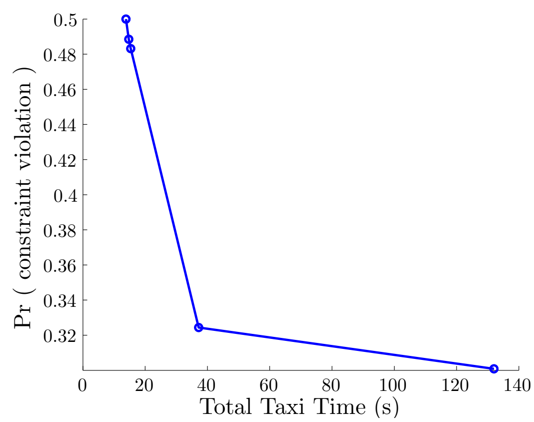

Figure 6. Objective function values for which any decrease in total taxi time results in an increase in the probability of constraint violation κ. If flights are required to be spaced cautiously, taxi time may increase as a result, sometimes significantly due to the varying lengths of possible paths.

As in this example, the choice of paths can make the total taxi time extremely sensitive to the imposed κ. Operators might use this Pareto front as a guide for determining a realistic expectation for taxi times based on a required κ, or vice versa. Here, a realistically-achievable value of κ might be approximately 33%.

Policy analysis example

In many circumstances the optimal solution for a given taxiway consists of patterns of paths that are common across varying schedules. Should our program often choose a particular set of trajectories, this might indicate the need for a taxi policy in which particular arcs are dedicated as one-way or exclusively for arrivals or departures. In the following example, a simple grid-based taxiway demonstrates the respective common paths chosen by departures and arrivals across many different schedules and starting/destination nodes. The only commonality of the trials is the taxiway layout and a general North-South direction for arrivals and departures.

We consider a taxiway composed of 9 nodes arranged in a 3×3 symmetrical grid that is complicated by an initial random perturbation of node positions to form an uneven grid (Figure 7a). Then for each of 1000 repetitions, we create a random number $n_{\text{dep}} \in \{1, 2, 3\}$ of departures that must traverse from randomly assigned top nodes to randomly assigned open bottom nodes of the grid. Meanwhile $n_{\text{arr}} = 3 - n_{\text{dep}}$ arrivals must traverse from bottom to top. Requirements for scheduling (Equation 2) in each run are sampled from a uniform distribution. Figure 7b shows the common patterns across all schedules. For example, the optimal choice of paths results in a pattern where arrivals are more likely to travel north via arc #8 than departures, and departures are more likely to travel south via arc #4 than arriving flights.

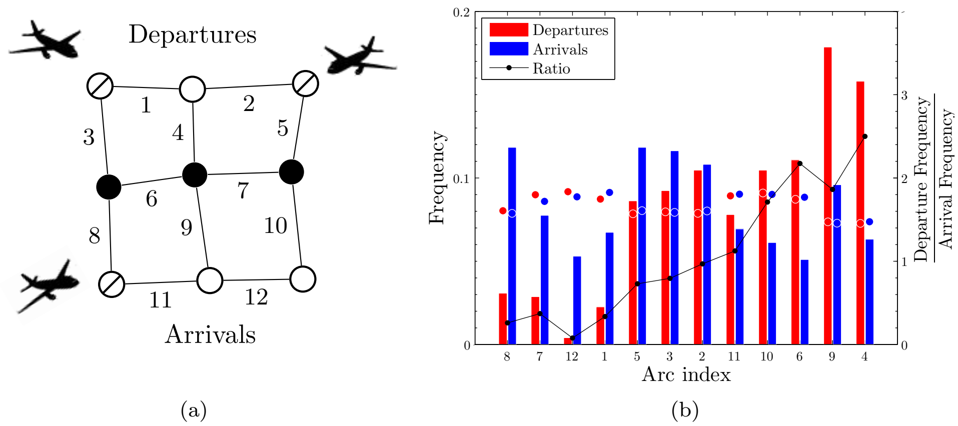

Figure 7. Frequency of arc traversal for departing and arriving flights on the taxiway in (a). For comparison, circles denote the frequency of each arc in the set $\mathcal{P}$ of departures and arrivals. Solutions for departing flights often consist of paths that are infrequently chosen by arriving flights, suggesting a policy in which these paths are designated solely for departing aircraft.

As the program is highly configurable, testing of current taxiway policies in large-scale scenarios is possible by fixing specific z_{ijk} and $B_{k_\alpha k_\beta i}$ variables.

V. Discussion

This paper presents a novel method to couple an aircraft scheduling problem to a trajectory planning problem constrained to a graph topology. The formulation is constructed in the time domain in order to consider flows of aircraft over the taxiway instead of node occupation. Unlike previous works that only consider shortest paths, the entire taxiway is included in the set of admissible solutions. Aircraft are prohibited from coming within specific spatial and temporal distances of one another, greatly increasing computational complexity. However if speeds across each arc are assumed to be constant, a mixed integer linear program developed herein is capable of tackling the problem within a reasonable time frame. By minimizing total taxi time and decreasing the probability of conflict among aircraft, the capacity of an airport taxiway is increased. These benefits are more noticeable with increased congestion.

Although focus is usually placed on runway positioning and orientation during airport design, a taxiway design that increases airport capacity is desirable, and use of this formulation as a discovery or analysis tool of taxiway layouts or traversal policies may aide this type of effort. Implementation in an airport setting either as a policy analysis tool or a real-time scheduling tool will alleviate some of the environmental and financial issues associated with taxiway queues and idling engines.

This approach may also be applicable to other control problems in multi-agent path planning, e.g., robotic swarms with uncertain dynamics, although a decrease in computational complexity may be necessary before this formulation is viable in real-time. Along these lines, possible modifications to the formulation include a constraint generation-based solution method (e.g., Benders' decomposition) or a quadratic program that avoids enumeration of paths.

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